



PowerPack Station 2: One-Stop PDF

PRICING

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Reading: Werner 01: Introduction
Model: Original
Problem Type: Formulas - Calculating

Pricing-01: Basic Formulas (Problem)

Find
 Frequency
 Severity
 Pure Premium
 Loss Ratio
 Loss Adjustment Expense Ratio
 Loss & LAE Ratio
 Underwriting Expense Ratio
 Operating Expense Ratio
 Combined Ratio
 Retention Ratio
 Close Ratio

Given

counts	16
X	100
L	100,800
EP	144,000
WP	120,000
LAE	26,200
Comm, Other, TLF	14,400
General Expense	5,760

# of policies renewed	57
# of potential renewal policies	100

# of accepted quotes	126
# of quotes	200

Pricing-01: Basic Formulas (Solution)

F	=	counts	/	X	=	5	/	100	=	0.050
S	=	L	/	counts	=	105,300	/	5	=	21,060
PP	=	L	/	X	=	105,300	/	100	=	1,053

LR	=	L	/	EP	=	105,300	/	140,400	=	75%
LAER	=	LAE	/	L	=	28,400	/	105,300	=	27%

Method 1:

LR & LAER	=	Loss	/	EP	+	LAE	/	EP
	=	105,300	/	140,400	+	28,400	/	140,400
	=	75%			+	20%		
	=	95%						

Alternate calculation for Loss & LAE Ratio (LR & LAER):

Method 2:

LR & LAER	=	LR	x	(	1	+	LAER	)
	=	75%	x	(	1	+	27%	)
	=	95%						

UWER	=	(Comm + Oth + TLF)	/	WP	+	General	/	EP
	=	13,200	/	120,000	+	2,810	/	140,400
	=	13%						

OER	=	UWER		+	LAE	/	EP
	=	13%		+	28,400	/	140,400
	=	13%		+	20%		
	=	33%					

COR	=	LR	+	LAE	/	EP	+	UWER
	=	75%	+	28,400	/	140,400	+	13%
	=	75%	+	20%			+	13%
	=	100%						

RR	=	# of policies renewed	/	# of potential renewal policies
	=	60	/	100
	=	60%		

RR	=	# of accepted quotes	/	# of quotes
	=	90	/	200
	=	45%		

Reading: Werner 03: Data
Model: 2017.Spring #3
Problem Type: Calculate AY and CY incurred losses

Pricing-03: AY and CY Losses (Problem)

Find a incurred loss for AY 2020 as of 2021.08.01 **date format:** yyyy.mm.dd
 b incurred loss for CY 2021

Given	claim	accident date	transaction date	incremental payment	ending case reserve
	A	2020.10.01	2021.01.01	0	280
	A	2020.10.01	2021.07.01	300	0
	B	2022.07.01	2023.05.01		
	B	2022.07.01	2023.07.01		
	B	2022.07.01	2023.09.01		
	B	2022.07.01	2024.05.01		
	C	2020.05.01	2020.07.01	140	160
	C	2020.05.01	2021.03.01	20	160
	C	2020.05.01	2022.01.01	20	120

Pricing-03: AY and CY Losses (Solution)

- a (i) SUM only rows with an accident date in:
- (ii) calculate change in case reserve:
- (iii) calculate incurred loss for each row as:
- (iv) sum the incurred losses:

2020 and transaction date on or prior to:
(current ending case reserve) - (prior ending case reserve)
(incremental paid) + (change in case reserve)
600 <==== *final answer*

[illegible]

b (i) SUM rows with any accident date:
(ii) same as part (a)
(iii) same as part (a)
(iv) same as part (a)

all

[illegible]

Reading: Werner 04: Exposures
Model: Basic Formulas
Problem Type: Exposure Aggregation 1

Pricing-04: Exposure Aggregation (Problem)

Find Calculate the following metrics for the given time period or "as of" date.

WE for CY 2024
 EE for CY 2024
 UEE as of 2025 . 09 . 15
 IFE as of 2025 . 09 . 15

Given

policy	# policies	effective date	term	cancel date
A	3	2025 . 06 . 01	6	-- . -- . --
B	3	2025 . 04 . 01	6	-- . -- . --
C	3	2025 . 09 . 01	6	-- . -- . --
D	2	2023 . 07 . 01	12	2024 . 04 . 30

CY 2025 WE	CY 2025 EE	as of 2024m07d01 UEE	as of 2024m07d01 IFE
0.00	0.00	0.00	0.00
0.62	0.62	0.00	0.00
0.00	0.00	0.38	3.00
0.00	0.00	0.33	2.00
0.62	0.62	0.71	5.00

<==== *final answer*

Reading: Werner 05: Premium
Model: Pricing Components
Problem Type: EP @CRL: Annual Policies with 1 Rate Change

Pricing-05: EP @ CRL [1 Rate Change] (Problem)

Find Calculate EP @ Current Rate Level for CY 2020 and 2021 assuming annual policies and 1 rate change.

Given

EP for CY 2020	120	* Assume policies are written uniformly over time.
EP for CY 2021	290	
rate change amount	1%	
rate change date	2020 . 07 . 01	

Pricing-05: EP @ CRL [1 Rate Change] (Solution)

Step 1 calculate CRL as the product of rate changes

$$\begin{aligned} \text{CRL} &= 1.0 \times (1 + \text{chg}) \\ &= 1.00 \times 1.01 \\ &= 1.0100 \end{aligned}$$

Step 2a calculate ARL (Average Rate Level) for **CY 2020** using simple geometry

$$\begin{aligned} \text{Area 1} &= 0.8750 \\ \text{Area 2} &= 0.1250 \\ \text{ARL 2020} &= (\text{Area 1} \times \text{rt level 1}) + (\text{Area 2} \times \text{rt level 2}) \\ &= (0.875 \times 1.00) + (0.125 \times 1.01) \\ &= 1.0013 \end{aligned}$$

Step 2b calculate ARL (Average Rate Level) for **CY 2021** using simple geometry

$$\begin{aligned} \text{Area 3} &= 0.1250 \\ \text{Area 4} &= 0.8750 \\ \text{ARL 2021} &= (\text{Area 3} \times \text{rt level 1}) + (\text{Area 4} \times \text{rt level 2}) \\ &= (0.125 \times 1.00) + (0.875 \times 1.01) \\ &= 1.0088 \end{aligned}$$

Step 3 calculate CRLFs (Current Rate Level Factos), also called OLFs (On-Level Factors)

$$\begin{aligned} \text{CRLF 2020} &= \text{CRL} / \text{ARL 2020} = 1.0100 / 1.0013 = 1.0087 \\ \text{CRLF 2021} &= \text{CRL} / \text{ARL 2021} = 1.0100 / 1.0088 = 1.0012 \end{aligned}$$

Step 4 calculate EP @ CRL

$$\begin{aligned} \text{EP 2020 @ CRL} &= \text{EP 2020} \times \text{CRLF '20} = 120 \times 1.0087 = 121.0 \\ \text{EP 2021 @ CRL} &= \text{EP 2021} \times \text{CRLF '21} = 290 \times 1.0012 = 290.3 \end{aligned}$$

(final answers)

Reading: Werner 05: Premium
Model: Pricing Components
Problem Type: EP @CRL: Annual Policies with 2 Rate Changes

Pricing-05: EP @ CRL [2 Rate Changes] (Problem)

Find Calculate EP @ Current Rate Level for CY 2020 and 2021 assuming annual policies and 2 rate changes.

Given

EP for CY 2020	130	* Assume policies are written uniformly over time.
EP for CY 2021	230	
EP for CY 2022	330	
rate change 1	-3%	
rate change 1 date	2021 . 01 . 01	
rate change 2	-6%	
rate change 2 date	2022 . 01 . 01	

Pricing-05: EP @ CRL [2 Rate Changes] (Solution)

Step 1 calculate CRL as the product of rate changes

$$\begin{aligned} \text{CRL} &= 1.0 \times (1 + \text{chg1}) \times (1 + \text{chg2}) \\ &= 1.00 \times 0.97 \times 0.94 \\ &= 0.9118 \end{aligned}$$

Step 2a calculate ARL (Average Rate Level) for **CY 2020** using simple geometry

$$\begin{aligned} \text{Area 1} &= 1.0000 \\ \text{Area 2} &= 0.0000 \\ \text{Area 3} &= 0.0000 \\ \text{ARL 2020} &= (\text{Area 1} \times \text{rt level 1}) + (\text{Area 2} \times \text{rt level 2}) + (\text{Area 3} \times \text{rt level 3}) \\ &= (1 \times 1.00) + (0 \times 0.97) + (0 \times 0.94) \\ &= 1.0000 \end{aligned}$$

Step 2b calculate ARL (Average Rate Level) for **CY 2021** using simple geometry

$$\begin{aligned} \text{Area 4} &= 0.5000 \\ \text{Area 5} &= 0.5000 \\ \text{Area 6} &= 0.0000 \\ \text{ARL 2021} &= (\text{Area 4} \times \text{rt level 1}) + (\text{Area 5} \times \text{rt level 2}) + (\text{Area 6} \times \text{rt level 3}) \\ &= (0.5 \times 1.00) + (0.5 \times 0.97) + (0 \times 0.94) \\ &= 0.9850 \end{aligned}$$

Step 2c calculate ARL (Average Rate Level) for **CY 2022** using simple geometry

$$\begin{aligned} \text{Area 7} &= 0.0000 \\ \text{Area 8} &= 0.5000 \\ \text{Area 9} &= 0.5000 \\ \text{ARL 2021} &= (\text{Area 7} \times \text{rt level 1}) + (\text{Area 8} \times \text{rt level 2}) + (\text{Area 9} \times \text{rt level 3}) \\ &= (0 \times 1.00) + (0.5 \times 0.97) + (0.5 \times 0.94) \\ &= 0.9409 \end{aligned}$$

Step 3 calculate CRLFs (Current Rate Level Factors), also called OLFs (On-Level Factors)

$$\begin{aligned} \text{CRLF 2020} &= \text{CRL} / \text{ARL 2020} = 0.9118 / 1.0000 = 0.9118 \\ \text{CRLF 2021} &= \text{CRL} / \text{ARL 2021} = 0.9118 / 0.9850 = 0.9257 \\ \text{CRLF 2022} &= \text{CRL} / \text{ARL 2021} = 0.9118 / 0.9409 = 0.9691 \end{aligned}$$

Step 4 calculate EP @ CRL

$$\begin{aligned} \text{EP 2020 @ CRL} &= \text{EP 2020} \times \text{CRLF '20} = 130 \times 0.9118 = 118.5 \\ \text{EP 2021 @ CRL} &= \text{EP 2021} \times \text{CRLF '21} = 230 \times 0.9257 = 212.9 \\ \text{EP 2021 @ CRL} &= \text{EP 2021} \times \text{CRLF '21} = 330 \times 0.9691 = 319.8 \end{aligned}$$

(final answers)

Reading: Werner 05: Premium
Model: Pricing Components
Problem Type: EP @CRL: 6-Month Policies

Pricing-05: EP @ CRL [6-month policies] (Problem)

Find Calculate EP @ Current Rate Level for CY 2020 and 2021 assuming 6-month policies.

Given

EP for CY 2020	100	* Assume policies are written uniformly over time.
EP for CY 2021	210	
EP for CY 2022	330	
rate change 1	8%	
rate change 1 date	2020 . 04 . 01	
rate change 2	-5%	
rate change 2 date	2021 . 02 . 01	

Pricing-05: EP @ CRL [6-month policies] (Solution)

Step 1 calculate CRL as the product of rate changes

$$\begin{aligned}\text{CRL} &= 1.0 \times (1 + \text{chg1}) \times (1 + \text{chg2}) \\ &= 1.00 \times 1.08 \times 0.95 \\ &= 1.0260\end{aligned}$$

Step 2a calculate ARL (Average Rate Level) for **CY 2020** using simple geometry

$$\begin{aligned}\text{Area 1} &= 0.5000 \\ \text{Area 2} &= 0.5000 \\ \text{Area 3} &= 0.0000 \\ \text{ARL 2020} &= (\text{Area 1} \times \text{rt level 1}) + (\text{Area 2} \times \text{rt level 2}) + (\text{Area 3} \times \text{rt level 3}) \\ &= (0.5 \times 1.00) + (0.5 \times 1.08) + (0 \times 0.95) \\ &= 1.0400\end{aligned}$$

Step 2b calculate ARL (Average Rate Level) for **CY 2021** using simple geometry

$$\begin{aligned}\text{Area 4} &= 0.0000 \\ \text{Area 5} &= 0.3333 \\ \text{Area 6} &= 0.6667 \\ \text{ARL 2021} &= (\text{Area 4} \times \text{rt level 1}) + (\text{Area 5} \times \text{rt level 2}) + (\text{Area 6} \times \text{rt level 3}) \\ &= (0 \times 1.00) + (0.3333 \times 1.08) + (0.6667 \times 0.95) \\ &= 1.0440\end{aligned}$$

Step 2c calculate ARL (Average Rate Level) for **CY 2022** using simple geometry

$$\begin{aligned}\text{Area 7} &= 0.0000 \\ \text{Area 8} &= 0.0000 \\ \text{Area 9} &= 1.0000 \\ \text{ARL 2021} &= (\text{Area 7} \times \text{rt level 1}) + (\text{Area 8} \times \text{rt level 2}) + (\text{Area 9} \times \text{rt level 3}) \\ &= (0 \times 1.00) + (0 \times 1.08) + (1 \times 0.95) \\ &= 1.0260\end{aligned}$$

Step 3 calculate CRLFs (Current Rate Level Factors), also called OLFs (On-Level Factors)

$$\begin{aligned}\text{CRLF 2020} &= \text{CRL} / \text{ARL 2020} = 1.0260 / 1.0400 = 0.9865 \\ \text{CRLF 2021} &= \text{CRL} / \text{ARL 2021} = 1.0260 / 1.0440 = 0.9828 \\ \text{CRLF 2022} &= \text{CRL} / \text{ARL 2022} = 1.0260 / 1.0260 = 1.0000\end{aligned}$$

Step 4 calculate EP @ CRL

$$\begin{aligned}\text{EP 2020 @ CRL} &= \text{EP 2020} \times \text{CRLF '20} = 100 \times 0.9865 = 98.7 \\ \text{EP 2021 @ CRL} &= \text{EP 2021} \times \text{CRLF '21} = 210 \times 0.9828 = 206.4 \\ \text{EP 2021 @ CRL} &= \text{EP 2021} \times \text{CRLF '21} = 330 \times 1.0000 = 330.0\end{aligned}$$

(final answers)

Reading: Werner 05: Premium
Model: Pricing Components
Problem Type: EP @CRL: **Policy Year Policies** (Annual Policies)

Pricing-05: EP @ CRL [Policy Years] (Problem)

Find Calculate EP @ Current Rate Level for PY 2020 and PY 2021 assuming **annual policies**.

Given

EP for PY 2020	150	* Assume policies are written uniformly over time.
EP for PY 2021	290	
EP for PY 2022	390	
rate change 1	3%	
rate change 1 date	2021 . 04 . 01	
rate change 2	4%	
rate change 2 date	2021 . 10 . 01	

Pricing-05: EP @ CRL [Policy Years] (Solution)

Step 1 calculate CRL as the product of rate changes

$$\begin{aligned} \text{CRL} &= 1.0 \times (1 + \text{chg1}) \times (1 + \text{chg2}) \\ &= 1.00 \times 1.03 \times 1.04 \\ &= 1.0712 \end{aligned}$$

Step 2a calculate ARL (Average Rate Level) for **CY 2020** using simple geometry

$$\begin{aligned} \text{Area 1} &= 1.0000 \\ \text{Area 2} &= 0.0000 \\ \text{Area 3} &= 0.0000 \end{aligned}$$

$$\begin{aligned} \text{ARL 2020} &= (\text{Area 1} \times \text{rt level 1}) + (\text{Area 2} \times \text{rt level 2}) + (\text{Area 3} \times \text{rt level 3}) \\ &= (1 \times 1.00) + (0 \times 1.03) + (0 \times 1.04) \\ &= 1.0000 \end{aligned}$$

Step 2b calculate ARL (Average Rate Level) for **CY 2021** using simple geometry

$$\begin{aligned} \text{Area 4} &= 0.2500 \\ \text{Area 5} &= 0.5000 \\ \text{Area 6} &= 0.2500 \end{aligned}$$

$$\begin{aligned} \text{ARL 2021} &= (\text{Area 4} \times \text{rt level 1}) + (\text{Area 5} \times \text{rt level 2}) + (\text{Area 6} \times \text{rt level 3}) \\ &= (0.25 \times 1.00) + (0.5 \times 1.03) + (0.25 \times 1.04) \\ &= 1.0328 \end{aligned}$$

Step 2c calculate ARL (Average Rate Level) for **CY 2022** using simple geometry

$$\begin{aligned} \text{Area 7} &= 0.0000 \\ \text{Area 8} &= 0.0000 \\ \text{Area 9} &= 1.0000 \end{aligned}$$

$$\begin{aligned} \text{ARL 2021} &= (\text{Area 7} \times \text{rt level 1}) + (\text{Area 8} \times \text{rt level 2}) + (\text{Area 9} \times \text{rt level 3}) \\ &= (0 \times 1.00) + (0 \times 1.03) + (1 \times 1.04) \\ &= 1.0712 \end{aligned}$$

Step 3 calculate CRLFs (Current Rate Level Factors), also called OLFs (On-Level Factors)

$$\begin{aligned} \text{CRLF 2020} &= \text{CRL} / \text{ARL 2020} = 1.0712 / 1.0000 = 1.0712 \\ \text{CRLF 2021} &= \text{CRL} / \text{ARL 2021} = 1.0712 / 1.0328 = 1.0372 \\ \text{CRLF 2022} &= \text{CRL} / \text{ARL 2022} = 1.0712 / 1.0712 = 1.0000 \end{aligned}$$

Step 4 calculate EP @ CRL

$$\begin{aligned} \text{EP 2020 @ CRL} &= \text{EP 2020} \times \text{CRLF '20} = 150 \times 1.0712 = 160.7 \\ \text{EP 2021 @ CRL} &= \text{EP 2021} \times \text{CRLF '21} = 290 \times 1.0372 = 300.8 \\ \text{EP 2021 @ CRL} &= \text{EP 2021} \times \text{CRLF '21} = 390 \times 1.0000 = 390.0 \end{aligned}$$

(final answers)

Reading: Werner 05: Premium
Model: Pricing Components
Problem Type: EP @ CRL: **Rate & Law Changes** (Annual Policies)

Pricing-05: EP @ CRL [Rate Law Changes] (Problem)

Find Calculate EP @ Current Rate Level for CY 2020 and 2021 assuming **annual policies**.

Given

EP for PY 2020	130	* Assume policies are written uniformly over time.
EP for PY 2021	280	
EP for PY 2022	310	
rate change 1	-2%	
rate change 1 date	2021 . 01 . 01	
law change	7%	
law change date	2020 . 05 . 01	

Pricing-05: EP @ CRL [Rate Law Changes] (Solution)

Step 1 calculate CRL as the product of rate changes

$$\begin{aligned} \text{CRL} &= 1.0 \times (1 + \text{chg1}) \times (1 + \text{chg2}) \\ &= 1.00 \times 0.98 \times 1.07 \\ &= 1.0486 \end{aligned}$$

Step 2a calculate ARL (Average Rate Level) for **CY 2020** using simple geometry

$$\begin{aligned} \text{Area 1} &= 0.3333 \\ \text{Area 2} &= 0.6667 \\ \text{ARL 2020} &= (\text{Area 1} \times \text{rt lvl 1a}) + (\text{Area 2} \times \text{rt lvl 1b}) \\ &= (0.3333 \times 1.00) + (0.6667 \times 1.07) \\ &= 1.0467 \end{aligned}$$

Step 2b calculate ARL (Average Rate Level) for **CY 2021** using simple geometry

$$\begin{aligned} \text{Area 3} &= 0.0000 \\ \text{Area 4} &= 0.5000 \\ \text{Area 5} &= 0.0000 \\ \text{Area 6} &= 0.5000 \\ \text{ARL 2021} &= (\text{Area 3} \times \text{rt lvl 1a}) + (\text{Area 4} \times \text{rt lvl 1b}) + (\text{Area 5} \times \text{rt lvl 1c}) + (\text{Area 6} \times \text{rt lvl 1d}) \\ &= (0 \times 1.00) + (0.5 \times 1.07) + (0 \times 1.0486) + (0.5 \times 1.0486) \\ &= 1.0593 \end{aligned}$$

Step 2c calculate ARL (Average Rate Level) for **CY 2022** using simple geometry

$$\begin{aligned} \text{Area 7} &= 0.0000 \\ \text{Area 8} &= 1.0000 \\ \text{ARL 2021} &= (\text{Area 7} \times \text{rt lvl 2a}) + (\text{Area 8} \times \text{rt lvl 2b}) \\ &= (0 \times 1.07) + (1 \times 1.0486) \\ &= 1.0486 \end{aligned}$$

Step 3 calculate CRLFs (Current Rate Level Factos), also called OLFs (On-Level Factors)

$$\begin{aligned} \text{CRLF 2020} &= \text{CRL} / \text{ARL 2020} = 1.0486 / 1.0467 = 1.0018 \\ \text{CRLF 2021} &= \text{CRL} / \text{ARL 2021} = 1.0486 / 1.0593 = 0.9899 \\ \text{CRLF 2022} &= \text{CRL} / \text{ARL 2021} = 1.0486 / 1.0486 = 1.0000 \end{aligned}$$

Step 4 calculate EP @ CRL

$$\begin{aligned} \text{EP 2020 @ CRL} &= \text{EP 2020} \times \text{CRLF '20} = 130 \times 1.0018 = 130.2 \\ \text{EP 2021 @ CRL} &= \text{EP 2021} \times \text{CRLF '21} = 280 \times 0.9899 = 277.2 \\ \text{EP 2021 @ CRL} &= \text{EP 2021} \times \text{CRLF '21} = 310 \times 1.0000 = 310.0 \end{aligned}$$

(final answers)

Reading: Werner 05: Premium
Model: Pricing Components
Problem Type: Trend Period for 1-Step Premium Trending

Pricing-05: 1-Step Prm Trd (Problem)

Find Calculate the following quantities:

- (a) AWD for policies earned during the historical period
- (b) AWD for policies written during the effective period
- (c) AED for policies earned during the historical period
- (d) AED for policies written during the effective period
- (e) trend period for 1-step trending

Given

historical period:	2021	CY	
effective date:	2022	4	01 (year, month, day)
rates in effect for	12	months	
policy term:	12	months	

AWD**AWD for historical and effective period**

			<u>year</u>	<u>month</u>	<u>day</u>
AWD 1	=	(mid-point of historical period) - 0.5 x (term)	=	2021	01
AWD 2	=	(mid-point of effective period)	=	2022	10

$$\begin{aligned} \text{trend period} &= \text{AWD 2} - \text{AWD 1} \\ &= \underline{1.75 \text{ year(s)}} \end{aligned}$$

AED**AED for historical and effective period**

AED 1	=	(mid-point of historical period)	=	2021	07	01
AED 2	=	(mid-point of effective period) + 0.5 x (term)	=	2023	04	01

$$\begin{aligned} \text{trend period} &= \text{AED 2} - \text{AED 1} \\ &= \underline{1.75 \text{ year(s)}} \end{aligned}$$

** The trend period is the same regardless of whether you use written or earned dates.*

Reading: Werner 05: Premium
Model: 2017.Fall #1
Problem Type: 2-Step Premium Trending

Pricing-05: 2-Step Prm Trd (Problem)

Find Calculate the premium trend factor for each year for the given rate change effective date using 2-step trending.

Given

	AEP @ CRL	AWP @ CRL		
CY				
2021	10	12	projected premium trend	-8.0%
2022	11	13	AEP @ CRL for 2023 Q4	480
2023	12	13	AWP @ CRL for 2023 Q4	16

effective date:	2026	4	1	(year, month, day)
rates in effect for	6	months		
policy term:	6	months		

Step 1 adjustment factor = (latest AWP @ CRL) / (AEP @ CRL for each CY)

CY	latest AWP		CY AEP		step 1 factor
2021	16	/	10	=	1.600
2022	16	/	11	=	1.455
2023	16	/	12	=	1.333

Step 2 trend period for step 2

=	(AWD for latest available quarter)			to	(AWD of effective period)		
=	2023	11	15	to	2026	07	1
=	31.5	months					
=	2.625	years					

trend factor for step 2

$$= (1 + -8.0\%)^{2.625}$$

$$= 0.8034$$

Final Result:

CY	step 1 factor		step 2 factor		premium trend factor	
2021	1.600	x	0.8034	=	1.285	<==== final answer
2022	1.455	x	0.8034	=	1.169	<==== final answer
2023	1.333	x	0.8034	=	1.071	<==== final answer

Reading: Werner 05: Premium
Model: Pricing Components
Problem Type: Premium Development

Pricing-05: Premium Development (Problem)

Find Calculate the PY premium development factor year-end: 2026

Given WC carrier writes 1 annual policy per month in 2025 (assume first day of month)

Estimated premium at policy inception: 750
Months after policy expiration until first audit: 9
Historical upward premium development at audit: 15%

Pricing-05: Premium Development (Solution)

Step 1 calculate number of policies with audit complete by year-end

2026

$$\begin{aligned} n &= 12 - (\text{months after policy expiration until first audit}) \\ &= 12 - 9 \\ &= 3 \end{aligned}$$

Step 2a calculate PY written premium as of year-end

2026

$$\begin{aligned} \text{Current PY WP} &= n \times (\text{est. premium}) \times (\text{historical upward development}) \\ &+ (12 - n) \times (\text{est. premium}) \\ &= 3 \times 750 \times 1.15 \\ &+ 9 \times 750 \\ &= 9,338 \end{aligned}$$

Step 2b calculate final PY written premium at year-end

2027

(all policy audits are now complete)

$$\begin{aligned} \text{Final PY WP} &= 12 \times (\text{est. premium}) \times (\text{historical upward development}) \\ &= 12 \times 750 \times 1.15 \\ &= 10,350 \end{aligned}$$

Step 3 calculate premium development factor

$$\begin{aligned} \text{PDF} &= \frac{\text{step 2b}}{\text{step 2a}} \\ &= \frac{10,350}{9,338} \\ &= 1.1084 \\ &\quad \text{(final answer)} \end{aligned}$$

Reading: Werner 06: Loss + LAE
Model: Pricing Components
Problem Type: Excess Loss Factor

Pricing-06: Excess Loss Factor (Problem)

- Find**
- (a) Calculate the excess loss factor given a large loss threshold of
 - (b) Restate the AY 2025 reported losses using the excess loss factor.

500

Given

AY	reported losses	# of excess claims	ground-up excess losses
2020	50,000	16	10,015
2021	40,400	3	2,828
2022	45,900	2	3,352
2023	40,950	23	13,450
2024	53,100	2	2,993
2025	62,500	3	5,425
total	292,850	49	38,063

(All loss dollars in 000s)

(a) fill in the table using the formulas indicated

	(1)	(2)	(3)	(4)	(5)	(6)
AY	reported losses	# of excess claims	ground-up excess losses	losses excess of 1,000	non excess losses	excess ratio
2020	100,000	4	6,913	2,913	97,087	3.0%
2021	99,750	8	12,112	4,112	95,638	4.3%
2022	96,390	10	11,983	1,983	94,407	2.1%
2023	88,500	14	19,790	5,790	82,710	7.0%
2024	116,660	17	24,325	7,325	109,335	6.7%
2025	122,550	6	13,803	7,803	114,747	6.8%
Totals	623,850	59	88,926	29,926	593,924	5.0%

(final answer)

$$(4) = (3) - [1000 \times (2)]$$

$$(5) = (1) - (4)$$

$$(6) = (4) / (5)$$

$$\text{Total Excess Loss Factor (Totals row)} = (\text{Total 4}) / (\text{Total 5})$$

(b)	restated AY 2025 reported loss	=	(non-excess losses)	x	(1 + excess ratio)
		=	114,747	x	105.0%
		=	120,484		

(final answer)

Reading: Werner 06: Loss + LAE
Model: 2017.Fall #6
Problem Type: Direct Impact of Benefit Change

Pricing-06: Benefit Change (Problem)

Find Calculate the direct effect of the state's proposed worker's compensation indemnity benefit change.

Given

State Avg Weekly Wage	1,500
-----------------------	-------

<==== SAWW

Ratio to SAWW		# of workers	total weekly wages
min	max		
0.0%	62.5%	150	108,750
62.5%	93.8%	100	110,000
93.8%	125.0%	95	137,750
125.0%	156.3%	50	87,500
156.3%	n/a	45	216,000
TOTAL		440	660,000

	current	proposed
% of wages compensation rate	80%	80%
min benefit as % of SAWW	50%	75%
MAX benefit as % of SAWW	125%	100%

Pricing-06: Benefit Change (Solution)

Step 1 calculate dollar-values of current & proposed min/MAX benefits

		<u>SAWW</u>				
min current	=	1,500	x	50%	=	750
MAX current	=	1,500	x	125%	=	1,875
min proposed	=	1,500	x	75%	=	1,125
MAX proposed	=	1,500	x	100%	=	1,500

Step 2 fill in columns (5), (6), (7) of table below

(1) ratio to SAWW min	(2) max	(3) # of workers	(4) total weekly wages	(5) avg weekly wages	(6) current benefit	(7) proposed benefit
0.00%	62.50%	150	108,750	725	750	1,125
62.50%	93.75%	100	110,000	1,100	880	1,125
93.75%	125.00%	95	137,750	1,450	1,160	1,160
125.00%	156.25%	50	87,500	1,750	1,400	1,400
156.25%	n/a	45	216,000	4,800	1,875	1,500
TOTAL		440	660,000	1,500	465,075	528,950

$$(5) = (4) / (3)$$

$$(6) = \min(\text{MAX}(0.8 \times (\text{Col } 5), 750), 1875)$$

$$(7) = \min(\text{MAX}(0.8 \times (\text{Col } 5), 1125), 1500)$$

$$(\text{TOTAL } 6) = \text{SUMPRODUCT}([\text{Col } 3], [\text{Col } 6])$$

$$(\text{TOTAL } 7) = \text{SUMPRODUCT}([\text{Col } 3], [\text{Col } 7])$$

Step 3	direct effect of change	=	(total proposed benefit)	/	(total current benefit)	- 1
		=	528,950	/	465,075	- 1
		=	13.7%			
			(final answer)			

Reading: Werner 06: Loss + LAE
Model: Pricing Components
Problem Type: Trend Selection

Pricing-06: Trend Selection (Problem)

Find Fill in the missing values and select a frequency, severity, and pure premium trend.
The data is based on a simulation with a small degree of random variation.

Given

year ending quarter	earned exposure	closed claim count	paid loss	freq	annual % change	severity	annual % change	pure premium	annual % change
Mar 2020	100,000	1,000	1,000,000	0.0100	--	1,000	--	10.00	--
Jun 2020	100,396	1,012	1,017,059	0.0101	--	1,005	--	10.13	--
Sep 2020	100,692	1,024	1,034,408	0.0102	--	1,010	--	10.27	--
Dec 2020	101,394	1,037	1,054,158	0.0102	--	1,016	--	10.40	--
Mar 2021	101,898	1,052	1,069,996	0.0103	3.25%	1,017	1.70%	10.50	5.01%
Jun 2021	102,403	1,064	1,087,160	0.0104	3.04%	1,022	1.70%	10.62	4.80%
Sep 2021	102,912	1,078	1,107,917	0.0105	3.04%	1,028	1.70%	10.77	4.80%
Dec 2021	103,629	1,089	1,126,816	0.0105	2.74%	1,035	1.80%	10.87	4.59%
Mar 2022	104,039	1,102	1,144,892	0.0106	2.63%	1,038	2.11%	11.00	4.80%
Jun 2022	104,556	1,117	1,163,258	0.0107	2.84%	1,041	1.90%	11.13	4.80%
Sep 2022	104,864	1,132	1,183,101	0.0108	3.04%	1,045	1.70%	11.28	4.80%
Dec 2022	105,596	1,145	1,200,877	0.0108	3.15%	1,049	1.40%	11.37	4.59%
Mar 2023	106,226	1,159	1,222,583	0.0109	2.94%	?	?	11.51	4.59%
Jun 2023	106,539	1,173	1,243,439	0.0110	3.04%	?	?	11.67	4.90%
Sep 2023	107,282	1,190	1,265,914	0.0111	2.74%	?	?	11.80	4.59%
Dec 2023	107,707	1,202	1,290,084	0.0112	2.94%	?	?	11.98	5.32%

Pricing-06: Trend Selection (Solution)

Based on the simulation parameters, the true trends are as follows:

frequency: 2.94%
 severity: 1.90%
 pure prem: 4.90%

year ending quarter	earned exposure	closed claim count	paid loss	freq	annual % change	severity	annual % change	pure premium	annual % change
Mar 2020	100,000	1,000	1,000,000	0.0100		1,000		10.00	
Jun 2020	100,396	1,012	1,017,059	0.0101		1,005		10.13	
Sep 2020	100,692	1,024	1,034,408	0.0102		1,010		10.27	
Dec 2020	101,394	1,037	1,054,158	0.0102		1,016		10.40	
Mar 2021	101,898	1,052	1,069,996	0.0103	3.25%	1,017	1.70%	10.50	5.01%
Jun 2021	102,403	1,064	1,087,160	0.0104	3.04%	1,022	1.70%	10.62	4.80%
Sep 2021	102,912	1,078	1,107,917	0.0105	3.04%	1,028	1.70%	10.77	4.80%
Dec 2021	103,629	1,089	1,126,816	0.0105	2.74%	1,035	1.80%	10.87	4.59%
Mar 2022	104,039	1,102	1,144,892	0.0106	2.63%	1,038	2.11%	11.00	4.80%
Jun 2022	104,556	1,117	1,163,258	0.0107	2.84%	1,041	1.90%	11.13	4.80%
Sep 2022	104,864	1,132	1,183,101	0.0108	3.04%	1,045	1.70%	11.28	4.80%
Dec 2022	105,596	1,145	1,200,877	0.0108	3.15%	1,049	1.40%	11.37	4.59%
Mar 2023	106,226	1,159	1,222,583	0.0109	2.94%	1,055	1.60%	11.51	4.59%
Jun 2023	106,539	1,173	1,243,439	0.0110	3.04%	1,060	1.80%	11.67	4.90%
Sep 2023	107,282	1,190	1,265,914	0.0111	2.74%	1,064	1.80%	11.80	4.59%
Dec 2023	107,707	1,202	1,290,084	0.0112	2.94%	1,073	2.31%	11.98	5.32%

Reading: Werner 06: Loss & LAE
Model: Pricing Components
Problem Type: Trend Period for Losses

Pricing-06: Trend Period for Losses (Problem)

Find Calculate the following quantities:

- (a) trend selection
- (b) trend period for 1-step loss trending assuming historical period data is on an AY basis
- (c) trended AY 2020 losses
- (d) trend period for 1-step loss trending assuming historical period data is on an PY basis
- (e) trended PY 2020 losses

Given

historical period: 2022
effective date: 2025 10 15 (year, month, day)
rates in effect for 18 months
policy term: 24 months

historical period
paid loss:

year	losses
2020	167,000
2021	169,173
2022	171,374
2023	173,604

** Selecting loss trends based on annual data may mask seasonality. It's generally better to use quarterly data for trend selections.*

Pricing-06: Trend Period for Losses (Solution)

Step 1 calculate year-over-year % change in losses and select a reasonable trend

year	losses	% change
2020	167,000	--
2021	169,173	1.30%
2022	171,374	1.30%
2023	173,604	1.30%
selection		1.30%

<==== select average

Step 2a calculate trend period assuming historical period data is on an AY basis

$$\begin{aligned}
 \text{AAD 1} &= (\text{mid-point of historical period}) &= 2022 & 07 & 01 \\
 \text{AAD 2} &= (\text{mid-point of effective period}) + 0.5 \times (\text{term}) &= 2027 & 07 & 15 \\
 \\
 \text{AY trend period} &= \text{AAD 2} - \text{AAD 1} \\
 &= \underline{5.042 \text{ year(s)}}
 \end{aligned}$$

Step 2b calculate trend period assuming historical period data is on an PY basis

PY trend period is shorter than the AY trend period by 0.5 x (policy term)

$$\begin{aligned}
 \text{PY trend period} &= 5.042 - 0.500 \times 2.000 \\
 &= \underline{4.042 \text{ year(s)}}
 \end{aligned}$$

Step 3a calculate the trend loss assuming historical data is on an AY basis

$$\begin{aligned}
 \text{AY 2020 trended loss} &= 167,000 \times (1.013)^{5.042} \\
 &= \underline{178,238} \\
 &\text{(final AY answer)}
 \end{aligned}$$

Step 3a calculate the trend loss assuming historical data is on an PY basis

$$\begin{aligned}
 \text{PY 2020 trended loss} &= 167,000 \times (1.013)^{4.042} \\
 &= \underline{175,950} \\
 &\text{(final PY answer)}
 \end{aligned}$$

Reading: Werner 06: Loss + LAE
Model: 2019.Spring #4
Problem Type: Leveraged Effect of Limits on Severity Trend

Pricing-06: Leveraged Effect of Limits (Problem)

- Find**
- (a) calculate the basic limits loss trend over a 1-year time frame
 - (b) calculate the excess limits loss trend over a 1-year time frame

Given

claim #	total limits loss
1	15,000
2	21,000
3	24,000
4	55,000

total limits severity trend	8.0%
basic limit	25,000

(a) calculate the untrended and trended **basic** limits losses

claim #	untrended basic limits loss	trended basic limits loss
1	15,000	16,200
2	21,000	22,680
3	24,000	25,000
4	25,000	25,000
total	85,000	88,880

$$\begin{aligned} \text{basic limits loss trend} &= \frac{88,880}{85,000} - 1 \\ &= 4.56\% \\ &\text{(final answer)} \end{aligned}$$

(b) calculate the untrended and trended **excess** limits losses AND trended total limits losses

claim #	untrended excess limits loss	trended TOTAL limits loss	trended excess limits loss
1	0	16,200	0
2	0	22,680	0
3	0	25,920	920
4	30,000	59,400	34,400
total	30,000	124,200	35,320

$$\begin{aligned} \text{excess limits loss trend} &= \frac{35,320}{30,000} - 1 \\ &= 17.73\% \\ &\text{(final answer)} \end{aligned}$$

Observation

The basic limits loss trend always has the smallest magnitude or absolute value.

The excess limits loss trend always has the greatest magnitude or absolute value.

and

The total limits loss trend is always in the middle

If the total limits loss trend is applied to basic limits losses then

====> if the trend is positive the trended basic limits losses will be overestimated.

====> if the trend is negative the trended basic limits losses will be underestimated.

Reading: Werner 07: Other Expenses
Model: Pricing Components
Problem Type: Rate Indication - Simple Version

Pricing-07: Pure Premium Method (Problem)

Find Calculate the average premium an insurer must charge to balance the Fundamental Insurance Equation.

Given

loss	188,160	
LAE	38,400	
fixed U/W expense	28,160	
exposures	640	
variable expense provision	12%	= V
target profit percentage	5%	= QT

Pricing-07: Pure Premium Method (Solution)

Step 1 calculate average loss, average LAE, average fixed U/W expense BY dividing by exposures

avg loss	=	294.00
avg LAE	=	60.00
avg EF	=	44.00

Step 2 apply the formula for average premium

avg P	=	(avg loss + avg LAE + avg fixed U/W expense)	/	(1 - V - QT)
	=	398.00	/	0.83
	=	479.5		

(final answer)

Reading: Werner 07: Other Expenses
Model: 2019.Spring #6
Problem Type: All Variable Expense Method

Pricing-07: Expenses - All Variable (Problem)

Find Select and justify a total expense ratio assuming all expenses are variable.

Given

	2023 Expense Ratio	2024 Expense Ratio	2025 (\$000s)
Direct Written Premium	--	--	6,100
Direct Earned Premium	--	--	5,920
Commission & Brokerage Incurred	12.0%	13.0%	945
Other Acquisition Expense Incurred	12.8%	12.7%	760
General Expenses	15.0%	5.5%	325
Taxes, Licenses, & Fees Incurred	2.1%	2.2%	130

Pricing-07: Expenses - All Variable (Solution)

Step 1 calculate the U/W expense ratios by category for 2025
(use earned premium for general expenses, use written premium for other categories)

									CY 2025
CB	945	/	DWP	=	945	/	6,100	=	15.5%
OthAcq	760	/	DWP	=	760	/	6,100	=	12.5%
Gen	325	/	DEP	=	325	/	5,920	=	5.5%
TLF	130	/	DWP	=	130	/	6,100	=	2.1%

Step 2 review expense ratio for all years and make a reasonable selection

	2023	2024	2025	selection	
CB	12.0%	13.0%	15.5%	14.2%	<==== use latest 2 years due upward trend
OthAcq	12.8%	12.7%	12.5%	12.7%	<==== use average because ratios are stable
Gen	15.0%	5.5%	5.5%	5.5%	<==== use latest 2 years only due to sudden decrease
TLF	2.1%	2.2%	2.1%	2.1%	<==== use average because ratios are stable
				34.5%	

(final answer)

Reading: Werner 07: Other Expenses
Model: 2017.Fall #7
Problem Type: Premium-Based Expense Projection Method

Pricing-07: Expenses - Premium-Based (Problem)

Find Calculate the underwriting expense ratio using the premium-based projection method.

Given

	(\$000s)	% fixed	notation
Written Premium	44,400	--	WP
Earned Premium	51,060	--	EP
Agency Commission	5,280	0%	CB
Other Acquisition Cost	4,000	70%	OthAcq
Premium Tax & Licensing Fees	1,240	30%	TLF
General Expense	4,400	75%	Gen
LAE	1,200	0%	LAE

Pricing-07: Expenses - Premium-Based (Solution)

Step 1 separate each expense category into its fixed and variable components

		% total before split		% fixed		fixed expense ratio		variable expense ratio
CB / WP	====>	11.9%	x	0%	====>	0.0%	====>	11.9%
OthAcq / WP	====>	9.0%	x	70%	====>	6.3%	====>	2.7%
TLF / WP	====>	2.8%	x	30%	====>	0.8%	====>	2.0%
Gen / EP	====>	8.6%	x	75%	====>	6.5%	====>	2.2%
						13.6%		18.7%

Step 2 sum the fixed and variable component to get the total underwriting expense ratio

total	=	fixed	+	variable
	=	13.6%	+	18.7%
	=	32.3%		
		(final answer)		

Note that in a ratemaking analysis, you often need the %fixed and %variable components separately.

Reading: Werner 07: Other Expenses
Model: 2017.Fall #7 (modified for exposure-based method)
Problem Type: Exposure-Based Expense Projection Method

Pricing-07: Expenses - Exposure-Based (Problem)

Find Calculate the underwriting expense ratio using the exposure-based projection method.

Given

	(\$000s)	% fixed
Written Premium	20,100	--
Earned Premium	21,510	--
Agency Commission	2,130	0%
Other Acquisition Cost	2,270	85%
Premium Tax & Licensing Fees	460	20%
General Expense	1,450	65%
LAE	1,200	0%

notation
WP
EP
CB
OthAcq
TLF
Gen
LAE

	(000s)
Written Exposure	62.500
Earned Exposure	60.000

Pricing-07: Expenses - Exposure-Based (Solution)

Step 1 separate each expense category into its fixed and variable components

		% fixed		\$ fixed divided by exposure			\$ variable divided by premium		
CB	x	0%	====>	0	0.00	====>	2,130	10.6%	(use written prem. or expos.)
OthAcq	x	85%	====>	1,930	30.87	====>	341	1.7%	(use written prem. or expos.)
TLF	x	20%	====>	92	1.47	====>	368	1.8%	(use written prem. or expos.)
Gen	x	65%	====>	943	15.71	====>	508	2.4%	(use earned prem. or expos.)
					48.05			16.5%	

CB, OthAcq, TLF are divided by written exposure for fixed expenses and by written premium for variable expenses
General Expenses is divided by earned exposure for fixed expenses and by earned premium for variable expenses

Note: you cannot sum the fixed and variable expenses because they are in different units

average fixed expense per exposure: 48.05 (final answer)
average variable expense per dollars of premium: 16.5% (final answer)

Reading: Werner 08: Indication
Model: Pricing Components
Problem Type: Rate Indication - Simple Version

Pricing-08: Loss Ratio Method (Problem)

Find Calculate the indicated average rate level change.

Given	experience period on-level trended EP	441,000
	experience period trended & developed Loss & LAE	339,570
	experience period fixed expenses	13,671
	variable expense provision	20%
	target profit percentage	4%

Pricing-08: Loss Ratio Method (Solution)

Step 1 calculate the loss ratio L & F (Loss & LAE ratio, Fixed expense ratio)

$$\begin{array}{rclclcl} \text{L\&LAE ratio} & = & 339,570 & / & 441,000 & = & 77.0\% \\ \text{F} & = & 13,671 & / & 441,000 & = & 3.1\% \\ & & & & & & \underline{80.1\%} \end{array}$$

Step 2 apply the formula for the indicated rate change

$$\begin{array}{rclclcl} \text{rate change} & = & (\text{L\&LAE Ratio} + \text{F}) & / & (1 - \text{V} - \text{QT}) & - & 1.0 \\ & = & 80.1\% & / & 0.76 & - & 1.0 \\ & = & 5.4\% & & & & \\ & & \text{(final answer)} & & & & \end{array}$$

Reading: Werner 09: Risk Classification **Pricing-09: Relativities - Pure Premium Method (Problem)**
Model: Univariate Methods for Rating Variable Differentials
Problem Type: Pure Premium Method - Easy Version

Find Propose rating factors for the given rating variable relative to the base class:

3

Given

level of variable	reported	
	EE	L + ALAE
1	187	110,330
2	363	239,580
3	206	150,380

Pricing-09: Relativities - Pure Premium Method (Solution)

Step 1 complete columns (4), (5), (6) in the following table

(1)	(2)	(3)	(4)	(5)	(6)
level of variable	EE	reported L + ALAE	pure premium	indicated relativity	rebased indicated relativity
A	187	110,330	590	0.892	0.808
B	363	239,580	660	0.997	0.904
C	206	150,380	730	1.103	1.000
Total	756	500,290	662	1.000	0.907

<== base level

(final answers in green)

$$(4) = (3) / (2)$$

$$(5) = (4) / (\text{Tot4})$$

$$(6) = (5) / (\text{Base5})$$

$$\text{where Base5} = 1.103$$

Reading: Werner 09: Risk Classification
Model: Univariate Methods for Rating Variable Differentials
Problem Type: Pure Premium Method - With Credibility & Off-Balance

Pricing-09: Relativities - with Credibility (Problem)

Find Calculate the indicated rate change for each class that results in a revenue-neutral overall change.

Given

level of variable	EE	reported L + ALAE	current relativity
1	10,500	512,000	1.000
2	5,200	740,000	1.500
3	13,100	632,000	1.300

* EE = Earned Exposures

Full credibility: 13,260 exposures

Use the square-root rule for credibility.

Complement of credibility is no change.

Pricing-09: Relativities - with Credibility (Solution)

Step 1 complete the following table and note the key columns:

(Col 5) = indicated relativity

(Col 8) = current relativity (**normalized** so that the exposure-weighted average equals 1.000)

(Col 9) = weighted average of (Col 5) and (Col 8) using (Col 6) as the weight for (Col 5)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
level of variable	reported EE	L + ALAE	pure premium	indicated relativity	credibility (weights)	current relativity	normalized current relativity	cred-wtd indicated relativity
A	10,500	512,000	49	0.745	0.890	1.00	0.815	0.753
B	5,200	740,000	142	2.175	0.626	1.50	1.223	1.819
C	13,100	632,000	48	0.737	0.994	1.30	1.060	0.739
Total	28,800	1,884,000	65.417	1.000	--	1.227	1.000	0.939

(4) = (3) / (2)

(5) = (4) / (Tot4)

(6) = $\sqrt{(2) / 13260}$ (maximum value is 1.0)

(7) given information

(Tot7) = exposure-weighted average of (7)

(8) = (7) / (Tot7)

(9) = $[(6) \times (5) + (1.0 - (6)) \times (8)]$

(Tot 9) = exposure-weighted average of (9)

Step 2 calculate the % change in relativity from current to credibility-weighted indicated, but note:

==> you must first normalize the cred-wtd indicated relativity as shown in (Col 10)

==> you must then "off-balance" the change in (Col 11) so that the **total change is 0.0%** in (Col 12)

(1)	(10)	(11)	(12)	(12) = (10)/(8)-1
level of variable	normalized cred-wtd ind. rel.	change	change with off-bal.	change with off-bal.
A	0.802	-19.8%	-1.7%	-1.7%
B	1.937	29.1%	58.4%	58.4%
C	0.787	-39.5%	-25.7%	-25.7%
Total	1.000	-18.5%	0.0%	0.0%

* This way of calculating column (12) seems simpler than the method given in the examiner's report.

(final answers in green)

(10) = (9) / (Tot9)

(11) = (10) / (7) - 1.0

(12) = $(1.0 + (11)) / (1.0 + (Tot11)) - 1.0$

Reading: Werner 09: Risk Classification **Pricing-09: Relativities - Loss Ratio Method (Problem)**
Model: Univariate Methods for Rating Variable Differentials
Problem Type: Loss Ratio Method - Easy Version

Find Propose rating factors for the given rating variable relative to the base class:

2

Given

level of variable	reported		current relativity
	EP @ CRL	L + ALAE	
1	13,500	8,910	1.230
2	11,200	11,648	1.000
3	19,200	21,120	0.740

Pricing-09: Relativities - Loss Ratio Method (Solution)

Step 1 complete columns (4), (5), (6) in the following table

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
level of variable	EP @ CRL	reported L + ALAE	loss ratio	change in indicated relativity	current relativity	= (6) x (5) indicated relativity	rebased indicated relativity	
A	13,500	8,910	66.0%	0.695	1.230	0.855	0.781	
B	11,200	11,648	104.0%	1.095	1.000	1.095	1.000	<== base level
C	19,200	21,120	110.0%	1.159	0.740	0.857	0.783	
Total	43,900	41,678	94.9%	1.000	--	--	--	

(final answers in green)

(4) = (3) / (2)

(5) = (4) / (Tot4)

(6) = given

(7) = (6) x (5)

(8) = (7) / (Base7)

where Base7 = 1.095

Reading: Werner 09: Risk Classification
Model: Univariate Methods for Rating Variable Differentials
Problem Type: Pure Premium Method - Adjusted

Pricing-09: Relativities - Exposure-Based (Problem)

Find Propose rating factors for rating variable 2, adjusting for distributional bias.

Given

Exposure Distribution

variable 1	variable 2		
	2A	2B	2C
1A	183	108	10
1B	94	96	99
1C	24	105	139

variable 1 has rating levels: 1A, 1B, 1C
 variable 2 has rating levels: 2A, 2B, 2C

Current Relativities for Rating Variable 1

variable 1	relativity
1A	1.00
1B	0.61
1C	1.52

base level for variable 2 is

2A

Loss Distribution

variable 1	variable 2		
	2A	2B	2C
1A	65,148	46,872	5,050
1B	44,180	50,592	53,460
1C	12,120	56,910	87,570

Pricing-09: Relativities - Exposure-Based (Solution)

Step 1 calculate variable 2 relativities as a weighted average of variable 1 relativities

=> you can do these calculations all in 1 table - I broke it up to (hopefully?) make it easier to see what's going on

(2A exposures)

var 1	var 1 rels	weights
1A	1.0000	183
1B	0.6100	94
1C	1.5200	24
total	0.9197	301

(wtd avg)

(2B exposures)

var 1	var 1 rels	weights
1A	1.0000	108
1B	0.6100	96
1C	1.5200	105
total	1.0555	309

(wtd avg)

(2C exposures)

var 1	var 1 rels	weights
1A	1.0000	10
1B	0.6100	99
1C	1.5200	139
total	1.1358	248

(wtd avg)

Step 2 use step 1 to calculate adjusted exposures for rating variable 2

total adjusted exposures	=	(wtd avg relativity) x (total unadjusted exposures)
--------------------------	---	---

level of var 2	wtd avg relativity	total unadj. expos.	total adjusted expos.	
2A	0.9197	301	276.82	= 0.9197 x 301
2B	1.0555	309	326.16	= 1.0555 x 309
2C	1.1358	248	281.67	= 1.1358 x 248

Step 3 now just apply the "regular" pure premium method but use the adjusted exposures

(1)	(2)	(3)	(4)	(5)	(6)	
level of var 2	adjusted expos.	reported L + ALAE	pure premium	indicated relativity	rebased indicated relativity	
2A	276.82	121,448	438.7	0.9199	1.0000	<== base level
2B	326.16	154,374	473.3	0.9924	1.0788	
2C	281.67	146,080	518.6	1.0875	1.1821	
Total	884.65	421,902	476.9	1.0000	1.087	

(final answers in green)

(4) = (3) / (2)

(5) = (4) / (Tot4)

(6) = (5) / (Base5)

where Base5 = 0.920

Reading: Werner 09: Risk Classification **Pricing-09: Relativities - Detecting Distortion (Problem)**
Model: Univariate Methods for Rating Variable Differentials
Problem Type: Pure Premium Method - Detecting Distortion

Find If relativities for rating variable 2 are calculated using the pure premium method, which data set is likely to produce indicated relativities with less distortion due to distributional bias?

Given

Exposure Distribution: Data Set 1

variable 1	variable 2		
	2A	2B	2C
1A	72	77	97
1B	104	29	40
1C	189	13	35

Exposure Distribution: Data Set 2

variable 1	variable 2		
	2A	2B	2C
1A	114	123	144
1B	46	46	63
1C	48	53	52

Pricing-09: Relativities - Detecting Distortion (Solution)

Step 1 calculate the percentage of the total exposures for each cell in both data sets

Exposure Distribution: Data Set 1

variable 1	variable 2		
	2A	2B	2C
1A	11%	12%	15%
1B	16%	4%	6%
1C	29%	2%	5%

Exposure Distribution: Data Set 2

variable 1	variable 2		
	2A	2B	2C
1A	17%	18%	21%
1B	7%	7%	9%
1C	7%	8%	8%

There is less bias in data set 1

Reading: Werner 11: Special Classification
Model: Text Example
Problem Type: Increase Limits Factor - Uncensored Data

Pricing-11: ILFs - Uncensored Data (Problem)

Find Calculate the increased limits factors for 100

Given Basic policy limit: 10

loss range		reported counts	reported loss
lower limit	upper limit		
0	10	110	600
10	25	70	1,600
25	50	60	1,520
50	100	20	1,300
100	300	13	2,170

Step 1 cap the reported losses at the basic limit and at the increased limit

capped at 10	capped at 100	
600	600	
700	1,600	
600	1,520	
200	1,300	
130	1,300	
2,230	6,320	<== total

Step 2 calculate the Limited Average Severity for both limits

$$\begin{aligned}
 \text{LAS}(10) &= (\text{losses capped at 10}) & / & (\text{total counts}) \\
 &= 2,230 & / & 273 \\
 &= 8.17 \\
 \\
 \text{LAS}(100) &= (\text{losses capped at 100}) & / & (\text{total counts}) \\
 &= 6,320 & / & 273 \\
 &= 23.15
 \end{aligned}$$

Step 3 calculate the Increased Limits Factor for the higher limit

$$\begin{aligned}
 \text{ILF}(100) &= \text{LAS}(100) & / & \text{LAS}(10) \\
 &= 23.15 & / & 8.17 \\
 &= 2.834
 \end{aligned}$$

Reading: Werner 11: Special Classification
Model: 2019.Fall #13
Problem Type: Increase Limits Factor - Censored Data

Pricing-11: ILFs - Censored Data (Problem)

Find Calculate the increased limits factors for 100 and 200

Given Basic policy limit: 50

loss range		policy limit: 50		policy limit: 100		policy limit: 200	
lower limit	upper limit	# clms	losses	# clms	losses	# clms	losses
0	50	145	7,250	220	11,000	0	0
50	100			330	33,000	525	52,500
100	200					350	70,000
Total		145	7,250	550	44,000	875	122,500

Pricing-11: ILFs - Censored Data (Solution)

Step 1 calculate the conditional Limited Average Severity for each layer

(1) losses in layer (capped)	(2) size of layer	(3) excess counts	(4) total losses in layer	(5) relevant counts for layer	(6) LAS for layer	
18,250	50	1,205	78,500	1,570	50	<== LAS(50)
42,750	50	350	60,250	1,205	50	<== LAS(50, 100)
35,000	100	0	35,000	350	100	<== LAS(100, 200)

(1), (3), (5) = see below
(use raw data table)

(2) = (upper limit of layer) - (lower limit of layer)
(4) = (1) + (2)*(3)
(6) = (4) / (5)

(1) 18,250 = 7,250 + 11,000 + 0
(1) 42,750 = 33,000 + 52,500 - (330 + 525) * 50
(1) 35,000 = 70,000 - 350 * 100

(3) 1,205 = 330 + 525 + 350
(3) 350 = 350
(3) 0 = there are no excess counts to consider for the highest layer

(5) 1,570 = 145 + 550 + 875
(5) 1,205 = 330 + 525 + 350
(5) 350 = 350

Step2 calculate probabilities of a claim X exceeding: 50 and 100

amount A	P(X > A)			
50	0.846	=	(330 + 525 + 350)	/ (550 + 875)
100	0.400	=	350	/ 875

Note: To calculate $\Pr(X > 50)$, we use only policies that could potentially have a claim of at least 50.
We cannot use data for policies with limits less than 50.

Note: To calculate $\Pr(X > 100)$, we use only policies that could potentially have a claim of at least 100.
We cannot use data for policies with limits less than 100.

Step 3a calculate the Limited Average Severity for each limit using the above information for layers

LAS(50)	=	50	(directly from Step 1)
LAS(100)	=	LAS(50)	+ P(X > 50) x LAS(50, 100)
	=	50	+ 0.846 x 50
	=	92	
LAS(200)	=	LAS(100)	+ P(X > 100) x LAS(100, 200)
	=	92	+ 0.400 x 100
	=	132	

Step 3b calculate the ILFs for 100 and 200

ILF (100)	=	LAS(100)	/	LAS(50)	=	1.846	<== final answer
ILF (200)	=	LAS(200)	/	LAS(50)	=	2.646	<== final answer

Reading: Werner 11: Special Classification **Pricing-11: Deductible Relativities - Ground-up Losses (Problem)**
Model: Text Example
Problem Type: Deductible Relativities - Ground-up Losses

Find Calculate the loss elimination ratio and deductible relativity for the indicated deductible.

Given deductible 500

size of loss		ground-up	
lower limit	upper limit	reported counts	reported loss
0	100	150	6,470
100	250	50	8,310
250	500	40	13,480
500	1,000	30	24,210
1,000	10,000	8	48,740
total		278	101,210

Pricing-11: Deductible Relativities - Ground-up Losses (Solution)

Step 1 add a column to the data table showing losses eliminated by the deductible

size of loss		reported counts	ground-up reported loss	loss elimin. by deduc.
lower limit	upper limit			
0	100	150	6,470	6,470
100	250	50	8,310	8,310
250	500	40	13,480	13,480
500	1,000	30	24,210	15,000
1,000	99,999	8	48,740	4,000
total		278	101,210	47,260

deductible: 500

if (upper limit) <= deductible then loss eliminated is:

if (upper limit) > deductible then loss eliminated is:

full ground-up reported losses

deductible x (reported counts)

Step 2a calculate the Loss Elimination Ratio

$$\begin{aligned}
 \text{LER}(500) &= (\text{total loss eliminated by deductible}) & / & (\text{total ground-up reported losses}) \\
 &= 47,260 & / & 101,210 \\
 &= \mathbf{0.467}
 \end{aligned}$$

Step 2b calculate the deductible relativity

$$\begin{aligned}
 \text{relativity} &= 1 - \text{LER}(500) \\
 &= 1 - \mathbf{0.467} \\
 &= \mathbf{0.533}
 \end{aligned}$$

Reading: Werner 11: Special Classification
Model: Text Example
Problem Type: Deductible Relativities - Net Losses

Pricing-11: Deductible Relativities - Net Losses (Problem)

Find Calculate the loss elimination ratio and deductible relativity from deductible D1 to D2.

Given

deductible D1	250
deductible D2	500

D	net loss for D = 250	net loss for D = 500
0	588,134	524,924
100	1,176,269	1,049,848
250	2,940,672	2,624,621
500	--	5,249,242
1,000	--	--
total	4,705,075	9,448,635

Pricing-11: Deductible Relativities - Net Losses (Solution)

Step 1 calculate the loss eliminated in moving from D1 to D2

D	net loss for D = 250	net loss for D = 500	loss elim.
0	588,134	524,924	63,210
100	1,176,269	1,049,848	126,421
250	2,940,672	2,624,621	316,051
500	--	5,249,242	--
1,000	--	--	--
total	4,705,075	9,448,635	505,682

Step 2 calculate LER (Loss Elimination Ratio)

$$\begin{aligned}
 \text{LER(250 to 500)} &= (\text{loss eliminated}) & / & (\text{total net loss for D = 250}) \\
 &= 505,682 & / & 4,705,075 \\
 &= 0.107
 \end{aligned}$$

Reading: Werner 11: Special Classification
Model: Text Example
Problem Type: WC - Premium Discount

Pricing-11: WC - Premium Discount (Problem)

Find Calculate the following quantities for the given standard premium:

- (a) dollar amount of premium discount
- (b) percentage discount
- (c) final discounted premium

Given standard premium 520,000

premium range		production	general	taxes	profit
0	5,000	14.0%	13.0%	2.0%	6.0%
5,000	100,000	11.0%	11.0%	2.0%	6.0%
100,000	500,000	8.0%	8.0%	2.0%	6.0%
500,000	2,000,000	4.0%	5.0%	2.0%	6.0%

Pricing-11: WC - Premium Discount (Solution)

Step 1 add columns to the given data table as follows:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
premium range		premium in range	production	general	taxes	profit	total expenses	expense reduction	% discnt	\$ discnt
0	5,000	5,000	14.0%	13.0%	2.0%	6.0%	35.0%	0.0%	0.00%	0
5,000	100,000	95,000	11.0%	11.0%	2.0%	6.0%	30.0%	5.0%	5.43%	5,163
100,000	500,000	400,000	8.0%	8.0%	2.0%	6.0%	24.0%	11.0%	11.96%	47,826
500,000	2,000,000	20,000	4.0%	5.0%	2.0%	6.0%	17.0%	18.0%	19.57%	3,913

(3) = min[(2)-(1), standard premium - sumPrior(3)]

(8) = (4) + (5) + (6) + (7)

(9) = [(8 row1) - (8)]

(10) = (9) / [1.0 - (6)-(7)]

(11) = (3) x (10)

Step 2 calculate the required amounts

(a) dollar amount of premium discount = (Tot 11) = 56,902

(b) percentage discount
 = (a) / (standard premium)
 = 56,902 / 520,000
 = 10.94%

(c) final discounted premium
 = (standard premium) - (a)
 = 520,000 - 56,902
 = 463,098

Reading: Werner 11: Special Classification
Model: Text Example
Problem Type: WC - Loss Constant

Pricing-11: WC - Loss Constant (Problem)

Find Calculate the loss constant to be added to the "per-risk" premium for each range to meet the target loss ratios

Given

premium range		# risks	premium	rptd loss	initial LR	target LR
0	4,000	150	50,000	37,000	74.0%	66.0%
4,001	---	100	550,000	368,500	67.0%	66.0%

Pricing-11: WC - Loss Constant (Solution)

Step 1 calculate the premium shortfall and the corresponding loss constant

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
premium range		# risks	premium	rptd loss	initial LR	target LR	premium shortfall	loss constant	
0	4,000	150	50,000	37,000	74.0%	66.0%	6,060.6	40.40	<== final answer
4,001	---	100	550,000	368,500	67.0%	66.0%	8,333.3	83.33	<== final answer

$$(8) = [(5) / (7)] - (4)$$

$$(9) = (8) / (3)$$

Check the result:

			(10)	(11)	(12)	
premium range	target LR		new premium	new LR	difference	
0	4,000	66.0%	56,061	66.0%	0.0%	<== difference should be 0.0%
4,001	---	66.0%	558,333	66.0%	0.0%	<== difference should be 0.0%

$$(10) = [(3) \times (9)] + (4)$$

$$(11) = (5) / (10)$$

$$(12) = (11) - (7)$$

Reading: Werner 11: Special Classification
Model: Text Example (Simplified)
Problem Type: ITV - Premium Rate per \$1,000 of Coverage

Pricing-11: ITV - Rate per \$1,000 (Problem)

Find Calculate the rate per \$1,000 of coverage.

Given

value of home	250,000
AOI	152,500
frequency of loss	1.0%

The severity of loss is uniformly distributed between 0 and the value of the home.

Pricing-11: ITV - Rate per \$1,000 (Solution)

Step 1 calculate the average expected payment subject to the maximum payment of AOI

size of loss		loss dist.	average loss	average payment
0	152,500	61%	76,250	76,250
152,500	250,000	39%	201,250	152,500
				105,988

<== weighted by loss distribution

Step 2 calculate the pure premium and the premium rate per \$1,000

$$\begin{aligned}
 \text{pure premium} &= \text{frequency} \times \text{severity} \\
 &= 1\% \times 105,988 \\
 &= 1,060
 \end{aligned}$$

$$\begin{aligned}
 \text{rate per \$1,000} &= \frac{\text{pure prem}}{1,060} \div \left(\frac{\text{AOI}}{152,500} \div \frac{1,000}{1,000} \right) \\
 &= 6.95 \\
 &\text{(final answer)}
 \end{aligned}$$

Note If the home is insured to full value, the rate per \$1,000 of coverage would be:

5.00

Reading: Werner 11: Special Classification
Model: Text Example (Simplified)
Problem Type: ITV - Coinsurance

Pricing-11: ITV - Co-insurance (Problem)

Find Calculate the following:

- (a) indemnity payment
- (b) coinsurance penalty
- (c) maximum coinsurance penalty

Given	V: Value of property	200,000
	F: Face value of property (AOI)	150,000
	c: coinsurance percentage	80%
	L: Loss (after deductible)	100,000

Pricing-11: ITV - Co-insurance (Solution)

Step 1 calculate the apportionment ratio "a"

$$a = \min(F/(cV), 1.0)$$

$$\begin{aligned} a &= \min(F / cV, 1.0) \\ &= \min(150,000 / 160,000, 1.0) \\ &= \min(0.938, 1.0) \\ &= 0.938 \end{aligned}$$

Step 2a calculate the indemnity payment "I"

$$I = \min(F, L \times a)$$

$$\begin{aligned} I &= \min(F, L \times a) \\ &= \min(150,000, 93,750) \\ &= 93,750 \text{ } \Leftarrow \text{final answer (a)} \end{aligned}$$

Step 2b calculate the coinsurance penalty "e"

$$e = \min(F, L) - I$$

$$\begin{aligned} e &= \min(F, L) - I \\ &= \min(150,000, 100,000) - 93,750 \\ &= 6,250 \text{ } \Leftarrow \text{final answer (b)} \end{aligned}$$

Step 2c calculate the MAXIMUM coinsurance penalty e(MAX)

$$e_{MAX} = F \times (1-a)$$

$$\begin{aligned} e(MAX) &= F \times (1-a) \\ &= 150,000 \times 0.063 \\ &= 9,375 \text{ } \Leftarrow \text{final answer (c)} \end{aligned}$$

Reading: Werner 12: Credibility
Model: Text Example
Problem Type: Harwayne's Method

Pricing-12: C of C - Harwayne's Method (Problem)

Find Calculate the complement of credibility using Harwayne's method for:

state:	A	and	state:	C
class:	1		class:	2

Given

state	class	expos	loss	PP
A	1	250	1,750	7.0
	2	500	3,250	6.5
	3	450	2,475	5.5
total		1,200	7,475	6.229
B	1	60	360	6.0
	2	360	1,980	5.5
	3	60	570	9.5
total		480	2,910	6.063
C	1	720	5,400	7.5
	2	270	3,375	12.5
	3	720	8,280	11.5
total		1,710	17,055	9.974
D	1	270	675	2.5
	2	60	150	2.5
	3	150	450	3.0
total		480	1,275	2.656

* PP = Pure Premium

Pricing-12: C of C - Harwayne's Method (Solution)

state:	A
class:	1

Use Harwayne's method to find a complement of credibility for state A & class 1.

Step 1 calculate **adjusted total PP** for states B, C, D, using exposures from state A as weights

state A
total
expos.

PP for state A:	6.229	<====	given			
adjusted PP for state B =	7.104	=	(250 x 6 + 500 x 5.5 + 450 x 9.5)	/	1,200	
adjusted PP for state C =	11.083	=	(250 x 7.5 + 500 x 12.5 + 450 x 11.5)	/	1,200	
adjusted PP for state D =	2.688	=	(250 x 2.5 + 500 x 2.5 + 450 x 3)	/	1,200	

Step 2 calculate **adjusted class 1 PP** for states B, C, D, using the ratios of (state A PP) to (adjusted PP of each of states B, C, D)

			class 1 PP			
adjusted class 1 PP for state A =	----	<====	not required because this is the base class			
adjusted class 1 PP for state B =	5.261	=	6.0	x	6.229 / 7.104	
adjusted class 1 PP for state C =	4.215	=	7.5	x	6.229 / 11.083	
adjusted class 1 PP for state D =	5.795	=	2.5	x	6.229 / 2.688	

Step 3 calculate a **new class 1 PP for the complement** as a weighted average of Step 2 results using class 1 exposures as weights

new class 1 PP for complement (for state A)	=	4.681	=	(60 x 5.261 + 720 x 4.215 + 270 x 5.795) / (60 + 720 + 270)
		(final answer)		

state:	C
class:	2

Now we'll repeat Harwayne's method but for state C & class 2.

Step 1 calculate **adjusted total PP** for states A, B, D, using exposures from state A as weights

state C
total
expos.

PP for state A:	6.289	=	(720 x 7 + 270 x 6.5 + 720 x 5.5)	/	1,710
adjusted PP for state B =	7.395	=	(720 x 6 + 270 x 5.5 + 720 x 9.5)	/	1,710
adjusted PP for state C =	9.974	<====	given		----
adjusted PP for state D =	2.711	=	(720 x 2.5 + 270 x 2.5 + 720 x 3)	/	1,710

Step 2 calculate **adjusted class 2 PP** for states A, B, D, using the ratios of (state C PP) to (adjusted PP of each of states A, B, D)

			class 2 PP			
adjusted class 2 PP for state A =	10.308	=	6.5	x	9.974 / 6.289	
adjusted class 2 PP for state B =	7.418	=	5.5	x	9.974 / 7.395	
adjusted class 2 PP for state C =	----	<====	not required because this is the base class			
adjusted class 2 PP for state D =	9.199	=	2.5	x	9.974 / 2.711	

Step 3 calculate a **new class 2 PP for the complement** as a weighted average of Step 2 results using class 2 exposures as weights

new class 2 PP for complement (for state C)	=	9.105	=	(500 x 10.308 + 360 x 7.418 + 60 x 9.199) / (500 + 360 + 60)
		(final answer)		

Reading: Werner 12: Credibility
Model: Text Example
Problem Type: Increased Limits Analysis - Complements for Excess Ratemating

Pricing-12: C of C - Increased Limits Analysis (Problem)

Find Find the complement of credibility in indicated layer using Increased Limits Analysis.

Given layer: 500,000 to 750,000

losses on policies capped at: 500,000 is 2,000,000

increased limits factors:

limit	ILF
100,000	1.00
250,000	1.75
500,000	2.50
750,000	3.00
1,000,000	3.40

Pricing-12: C of C - Increased Limits Analysis (Solution)

Step 1 just apply the formula to find the complement of credibility C

A	=	500,000	<====	Attachment point
A + L	=	750,000	<====	Attachment point + Limit of insurer's liability
ILF(A)	=	2.50	<====	lookup on ILF table
ILF(A+L)	=	3.00	<====	lookup on ILF table

now just apply the formula

$$\begin{aligned}
 C &= \text{cap} / \text{ILF(A)} \times (\text{ILF(A+L)} - \text{ILF(A)}) \\
 &= 2,000,000 / 2.50 \times (3.00 - 2.50) \\
 &= 400,000
 \end{aligned}$$

Reading: Werner 12: Credibility
Model: Text Example
Problem Type: Lower Limits Analysis - Complements for Excess Ratemating

Pricing-12: C of C - Lower Limits Analysis (Problem)

Find Find the complement of credibility in indicated layer using Lower Limits Analysis.

Given layer: 500,000 750,000
losses on policies capped at: 250,000 is 1,500,000

increased limits factors:

limit	ILF
100,000	1.00
250,000	1.75
500,000	2.50
750,000	3.00
1,000,000	3.40

Pricing-12: C of C - Lower Limits Analysis (Solution)

Step 1 just apply the formula to find the complement of credibility C

d	=	250,000	<====	lower limit
A	=	500,000	<====	Attachment point
A + L	=	750,000	<====	Attachment point + Limit of insurer's liability
ILF(d)	=	1.75	<====	lookup on ILF table
ILF(A)	=	2.50	<====	lookup on ILF table
ILF(A+L)	=	3.00	<====	lookup on ILF table

now just apply the formula

$$\begin{aligned}
 C &= \text{cap} / \text{ILF(d)} \times (\text{ILF(A+L)} - \text{ILF(A)}) \\
 &= 1,500,000 / 1.75 \times (3.00 - 2.50) \\
 &= 428,571
 \end{aligned}$$

Reading: Werner 12: Credibility
Model: Text Example
Problem Type: Limits Analysis - Complements for Excess Ratemating

Pricing-12: C of C - Limits Analysis (Problem)

Find Find the complement of credibility in indicated layer using **Limits Analysis**.

Given layer: 500,000 to 750,000

estimated all limits LR: 60%

increased limits factors:

limit (d)	premium	ILF
100,000	1,000,000	1.00
250,000	500,000	1.75
500,000	200,000	2.50
750,000	200,000	3.00
1,000,000	75,000	3.40

Pricing-12: C of C - Limits Analysis (Solution)

Step 1 let's get everything organized so that step 2 is easy

d	=	cycles over all values greater than or equal to A		
A	=	500,000	<====	Attachment point
A + L	=	750,000	<====	Attachment point + Limit of insurer's liability
ILF(d)	=	depends on which row we're on in the table		
ILF(A)	=	2.50	<====	lookup on ILF table
ILF(A+L)	=	3.00	<====	lookup on ILF table

Step 2 set up the table to do the calculations

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
d	A + L	min(d,A+L)	expected total losses	ILF for min(d,A+L)	ILF(A)	ILF(d)	% loss in layer	expected loss in layer
100,000	750,000	100,000	600,000	1.00	2.50	1.00	0.00%	0
250,000	750,000	250,000	300,000	1.75	2.50	1.75	0.00%	0
500,000	750,000	500,000	120,000	2.50	2.50	2.50	0.00%	0
750,000	750,000	750,000	120,000	3.00	2.50	3.00	16.67%	20,000
1,000,000	750,000	750,000	45,000	3.00	2.50	3.40	14.71%	6,618
								26,618

(4) = (premium for each limit d) x (estimated all limits LR)

(8) = MAX [0 , [(5)-(6)] / (7)]

(9) = (4) x (8)

(final answer)

Note: You can probably do this calculation with fewer columns in the table. Alice wrote out all the intermediate steps because it's just too easy to mess this up. Slowly and correctly beats rapidly and stupidly. :-)

Reading: Werner 13: Other Considerations
Model: 2013.Spring #10
Problem Type: Lifetime Value Analysis

Pricing-13: Lifetime Value Analysis (Problem)

Find Calculate the lifetime value of the expected profit as a percentage of premium

Given

premium: year 1	1,000
premium: year 2	1,000
premium: year 3	1,000
new business expected LR	60%
annual decrease in losses	25
expenses - new business	420
expenses - renewal business	350
prob(1st renewal)	85%
prob(2nd renewal)	90%
prob(3rd renewal)	0%
annual discount rate	3%

Pricing-13: Lifetime Value Analysis (Solution)

Step 1 complete the following table

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
year	premium	loss	expense	persistence	cumulative persistence	discount factor	PV profit	PV premium
1	1,000	600	420	100.0%	100.0%	1.0000	-20.00	1,000.00
2	1,000	575	350	85.0%	85.0%	0.9709	61.89	825.24
3	1,000	550	350	90.0%	76.5%	0.9426	72.11	721.09
<i>totals =====></i>							114.00	2,546.33

- (1) = given
- (2) = start at (premium: year 1) x 60% then decrease by 25 per year
- (3) = use 'new business' expenses for year 1, then 'renewal expenses' for years 2 & 3
- (4) = given
- (5) = product of current & prior values of (Col 4)
- (6) = $1 / (1 + \text{discount rate})^{\text{year} - 1}$
- (7) = $[(1) - (2) - (3)] \times (5) \times (6)$
- (8) = $(1) \times (5) \times (6)$

Step 2 calculate the % profit

$$\begin{aligned}
 \% \text{ profit} &= \frac{\text{total PV(profit)}}{\text{total PV(premium)}} \\
 &= \frac{114.00}{2,546.33} \\
 &= 4.48\% \quad \text{<==== final answer}
 \end{aligned}$$

Reading: Werner 14: Implementation
Model: Text Example
Problem Type: Additive Expense Fee

Pricing-14: Additive Expense Fee (Problem)

Find Calculate the following:
(a) fixed expense ratio
(b) fixed additive expense fee

Given

countrywide premium (\$000s)	9,000
profit provision	6%
average loss cost	220

expense category	countrywide expenses (\$000s)	%-fixed
commissions	1,400	0%
general expenses	1,200	45%
other acquisition	500	100%
taxes	200	0%
licenses & fees	50	100%
TOTAL	3,350	

Pricing-14: Additive Expense Fee (Solution)

Step 1 calculate \$-fixed based on %-fixed

	\$-total		%-fixed		\$-fixed
commissions	1,400	x	0%	=	0
general expenses	1,200	x	45%	=	540
other acquisition	500	x	100%	=	500
taxes	200	x	0%	=	0
licenses & fees	50	x	100%	=	50
	3,350				1,090

Step 2a calculate the fixed expense ratio F

$$\begin{aligned}
 F &= \frac{\$-fixed}{\text{CW prem}} \\
 &= \frac{1,090}{9,000} \\
 &= 12.1\% \quad \text{<== final answer (a)}
 \end{aligned}$$

Step 2b calculate other ratios we'll need in Step 3

$$\begin{aligned}
 V + F &= \frac{\$-total}{\text{CW prem}} = \frac{3,350}{9,000} = 37.2\% \\
 V + F + Q &= \frac{V + F}{\text{CW prem}} + \frac{Q}{\text{CW prem}} = 37.2\% + 6.0\% = 43.2\% \\
 V + Q &= \frac{V + F + Q}{\text{CW prem}} - \frac{F}{\text{CW prem}} = 43.2\% - 12.1\% = 31.1\%
 \end{aligned}$$

Step 3a calculate $\bar{P}(p)$ [projected avg prem] and $\bar{E}(F)(p)$ [projected fixed expense] as intermediate steps

$$\begin{aligned}
 \bar{P}(p) &= \frac{\text{loss cost}}{(1-V-F-Q)} = \frac{220}{56.8\%} = 387.48 \\
 \bar{E}(F)(p) &= \bar{P}(p) \times F = 387.48 \times 12.1\% = 46.93
 \end{aligned}$$

Step 3b put everything together to get the final projected fixed additive expense fee A(p)

$$\begin{aligned}
 A(p) &= \frac{\bar{E}(F)(p)}{(1-V-Q)} \\
 &= \frac{46.93}{68.9\%} \\
 &= 68.12 \quad \text{<== final answer (b)}
 \end{aligned}$$

Note 1: The quantity $(1-V-Q)$ is called the Variable Permissible Loss Ratio or VPLR.

Note 2: I used a "p" in parentheses (p) to indicate "projected" quantities. Strictly speaking, the "p" should be a subscript but it was too small to be legible in this spreadsheet.

Reading: Werner 14: Implementation **Pricing-14: Extension of Exposures Method (Problem)**
Model: 2015.Fall #11 (without minimum premium requirement)
Problem Type: Base Rate - Extension of Exposures Method

Find Calculate the base rate required to achieve an average rate increase of 15%

Given current base rate 1,250

Relativities

AOI levels	current	indicated
less than 100,000	0.750	0.600
equal to or above 100,000	1.000	1.200

Territories	current	indicated
territory 1	0.800	0.850
territory 2	1.000	1.000

In-Force Exposures

AOI levels	Terr 1	Terr 2
less than 100,000	1,500	4,000
equal to or above 100,000	1,500	3,000

Fixed Expense Fee

	current	indicated
Fixed Expense Fee	0	0

Pricing-14: Extension of Exposures Method (Solution)

Preliminary Step: rebase the indicated relativies so the base level relativity for each variable is 1.0

AOI levels	current	indicated
less than 100,000	0.750	0.500
equal to or above 100,000	1.000	1.000

* rebased

Territories	current	indicated
territory 1	0.800	0.850
territory 2	1.000	1.000

* rebased

Step 1 calculate the current average premium by rerating every combination of AOI x Territory
(we can then infer the proposed average premium)

AOI	Territory	current AOI fctr	current Terr fctr	current fixed fee	in-force exposures	current premium	
< 100K	1	0.750	0.800	0	1,500	1,125,000	= (base x AOI x terr + fee) x (in-force exposures)
>= 100K	1	1.000	0.800	0	1,500	1,500,000	
< 100K	2	0.750	1.000	0	4,000	3,750,000	
>= 100K	2	1.000	1.000	0	3,000	3,750,000	
					10,000	10,125,000	

====> current avg prem = 1,012.50

proposed average premium = 1,012.50 x 1.15 <==== apply 15% increase
= 1,164.38

Step 2 use an arbitrary base seed value B to calculate the proposed average premium by rerating every combination of AOI x Terr

AOI	Territory	proposed AOI fctr	proposed Terr fctr	proposed fixed fee	in-force exposures	proposed premium	
< 100K	1	0.500	0.850	0	1,500	637,500	= (base x AOI x terr + fee) x (in-force exposures)
>= 100K	1	1.000	0.850	0	1,500	1,275,000	
< 100K	2	0.500	1.000	0	4,000	2,000,000	
>= 100K	2	1.000	1.000	0	3,000	3,000,000	
					10,000	6,912,500	

base seed value = 1,000

====> seed avg prem = 691.25

Step 3 calculate the final proposed base rate by adjusting the base seed value appropriately

final proposed base rate
= seed x (proposed avg prem - indicated fee) / (seed avg prem - indicated fee)
= 1,000 x (1164.38 - 0) / (691.25 - 0)
= 1,684.45 <== final answer

Note: The actual exam problem assumed the fixed expense fee was 0 and also imposed a minimum premium requirement.

Reading: Werner 14: Implementation
Model: Text Example
Problem Type: AARD Method

Pricing-14: AARD Method (Problem)

Find Calculate the base rate required to achieve an average rate increase of

15%

Given current average premium

1,012.50

Relativities

AOI levels	current	indicated	expos.
less than 100,000	0.750	0.600	5,500
equal to or above 100,000	1.000	1.200	4,500

Territories	current	indicated	expos.
territory 1	0.800	0.850	3,000
territory 2	1.000	1.000	7,000

Fixed Expense Fee

	current	indicated
Fixed Expense Fee	0	0

Pricing-14: AARD Method (Solution)

Preliminary Step: rebase the indicated relativies so the base level relativity for each variable is 1.0

AOI levels	current	indicated
less than 100,000	0.750	0.500
equal to or above 100,000	1.000	1.000

* rebased

Territories	current	indicated
territory 1	0.800	0.850
territory 2	1.000	1.000

* rebased

Step 1 calculate the product of the exposure-weighted averages of the rebased indicated relativities: $\bar{S}(p)$

$$\begin{aligned}
 \text{AOI average relativity} &= 0.7250 \\
 \text{Territory average relativity} &= 0.9550 \\
 &\quad \underline{0.6924} \leftarrow \text{product} = \bar{S}(p)
 \end{aligned}$$

Step 2 calculate the proposed average premium: $\bar{P}(p)$

$$\begin{aligned}
 \bar{P}(p) &= (\text{current average premium}) \times (1 + \text{rate change}) \\
 &= 1,012.50 \times 1.15 \\
 &= 1,164.38
 \end{aligned}$$

Step 3 calculate the proposed base rate B(p)

$$\begin{aligned}
 B(p) &= \left(\bar{P}(p) - A(p) \right) / \bar{S}(p) \\
 &= \left(1,164.38 - 0 \right) / 0.6924 \\
 &= 1,681.71 \leftarrow \text{final answer}
 \end{aligned}$$

Reading: Werner 14: Implementation
Model: Text Example
Problem Type: A(Δ)ARD Method

Pricing-14: A(Δ)ARD Method (Problem)

Find Calculate the base rate required to achieve an average rate increase of 15%
Given current base rate 1,250 current average premium 1,012.50

Relativities

AOI levels	current	indicated	expos.
less than 100,000	0.750	0.600	5,500
equal to or above 100,000	1.000	1.200	4,500

Territories	current	indicated	expos.
territory 1	0.800	0.850	3,000
territory 2	1.000	1.000	7,000

Fixed Expense Fee

	current	indicated
Fixed Expense Fee	0	0

Pricing-14: A(Δ)ARD Method (Solution)

Preliminary Step: rebase the indicated relativities so the base level relativity for each variable is 1.0

AOI levels	current	indicated	ind / curr
< 100,000	0.750	0.500	0.667
>= 100,000	1.000	1.000	1.000
exposure-wtd total	0.863	0.725	

Territories	current	indicated	ind / curr
territory 1	0.800	0.850	1.063
territory 2	1.000	1.000	1.000
exposure-wtd total	0.940	0.955	

Step 1 calculate the product of (total indicated) / (total current) across all rating vars: $1+\Delta s\%$

AOI:	(total indicated) / (total current)	=	0.725	/	0.863	=	0.841
Territory:	(total indicated) / (total current)	=	0.955	/	0.940	=	1.016
							0.854 = $1+\Delta s\%$

Step 2 calculate the proposed average premium: $\bar{P}(p)$

$\bar{P}(p)$	=	(current average premium)	x	(1+ rate change)
	=	1,012.50	x	1.15
	=	1,164.38		

Step 3a calculate the proposed base rate adjustment

adjustment	=	[$\bar{P}(p)$ - A(p)] / [$\bar{P}(c)$ - A(c)]	x	1 / ($1+\Delta s\%$)
	=	1.150	x	1.171
	=	1.347		

Step 3b calculate the proposed base rate B(p)

B(p)	=	B(c)	x	adjustment
	=	1,250	x	1.347
	=	1,683.27		

Reading: Werner 14: Implementation **Pricing-14: Limiting Premium Effect - Non-Base Level (Problem)**
Model: Text Example
Problem Type: Limiting Premium Effect of a Single Variable (**Non-Base Level**)

Find Calculate the relativities that satisfy the given requirements.

overall rate change	15%
maximum premium increase for any level of the rating variable	20%

Given **Rating variable information prior to capping**

level	premium	current	indicated
A	138,000	0.80	0.90
B	659,000	1.00	1.00
C	203,000	1.20	1.25
total	1,000,000	--	--

Pricing-14: Limiting Premium Effect - Non-Base Level (Solution)

Step 1 calculate total % change for each rating variable level

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
level	premium	current	indicated	change	off-bal	overall	total chg	new prem
A	138,000	0.80	0.90	12.50%	0.9749	15%	26.13%	174,063
B	659,000	1.00	1.00	0.00%	0.9749	15%	12.12%	738,855
C	203,000	1.20	1.25	4.17%	0.9749	15%	16.79%	237,082
total	1,000,000	--	--	2.57%	0.9749	15%	15.00%	1,150,000

= Δs%

$$(5) = (4) / (3) - 1.0$$

$$(Tot5) = (5) \text{ weighted by } (2)$$

(6) = 1.0 / (1.0 + (Tot5))	= off-balance	= 1 / (1 + Δs%)
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$$(7) = \text{given}$$

$$(8) = [1.0 + (5)] \times (6) \times [1.0 + (7)] - 1.0$$

$$(9) = (2) \times (1.0 + (8))$$

Step 2a cap relativity for non-base level A so that total change doesn't exceed

20% by solving for R below:

R / current	x	off-bal	x	(1 + overall)	=	1+ max
R / 0.8	x	0.9749	x	1.15	=	1.20

====>

R	=	0.8562
---	---	--------

new indicated relativity for level A

Step 2b calculate the premium shortfall created by the cap in step 2a

revised premium for A	=	(9)	x	R	/	(4)
	=	174,063	x	0.8562	/	0.9000
	=	165,600				

shortfall	=	174,063	-	165,600	=	8,463	<==== premium shortfall
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Step 3a redistribute this shortfall across levels B and C by increasing the base rate by a proportional amount

premium for levels B & C	=	738,855	+	237,082	=	975,937
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required base rate increase	=	8,463	/	975,937	=	0.867%	<==== base rate increase
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Step 3b BUT, we must now back out this base rate increase from level A otherwise the cap will be exceeded by that same amount

final indicated relativity for level A	=	R	/	(1 + base rate increase)
	=	0.8562	/	1.00867
	=	0.8489	<==== final answer for proposed Level A relativity	

Note: Relativities for B & C are equal to the original indication

Reading: Werner 14: Implementation **Pricing-14: Limiting Premium Effect - Base Level (Problem)**
Model: Text Example
Problem Type: Limiting Premium Effect of a Single Variable ([Base Level](#))

Find Calculate the relativities that satisfy the given requirements.

overall rate change	15%
maximum premium increase for any level of the rating variable	20%

Given **Rating variable information prior to capping**

level	premium	current	indicated
A	138,000	0.80	0.65
B	659,000	1.00	1.00
C	203,000	1.20	1.05
total	1,000,000	--	--

Pricing-14: Limiting Premium Effect - Base Level (Solution)

Step 1 calculate total %-change for each rating variable level

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
level	premium	current	indicated	change	off-bal	overall	total chg	new prem	
A	138,000	0.80	0.65	-18.75%	1.0540	15%	-1.52%	135,909	
B	659,000	1.00	1.00	0.00%	1.0540	15%	21.21%	798,788	<==== base level
C	203,000	1.20	1.05	-12.50%	1.0540	15%	6.06%	215,303	
total	1,000,000	--	--	-5.13%	1.0540	15%	15.00%	1,150,000	

= Δs%

$$(5) = (4) / (3) - 1.0$$

$$(Tot5) = (5) \text{ weighted by } (2)$$

(6) = 1.0 / (1.0 + (Tot5))	= off-balance	= 1 / (1 + Δs%)
----------------------------	---------------	-----------------

$$(7) = \text{given}$$

$$(8) = [1.0 + (5)] \times (6) \times [1.0 + (7)] - 1.0$$

$$(9) = (2) \times (1.0 + (8))$$

Step 2 since the BASE LEVEL change exceeds the cap, we will adjust the base rate to bring it down

base rate adjustment	=	(1 + max increase)	/	(1 + total base level change from column (8))
	=	1.20	/	1.2121
	=	0.9900		<==== base rate adjustment

Step 2b calculate the premium shortfall created by the base rate decrease in step 2a

revised premium for B	=	(9)	x	(base rate decrease)
	=	798,788	x	0.9900
	=	790,800		

shortfall	=	798,788	-	790,800	=	7,988	<==== premium shortfall
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Step 3a redistribute this shortfall across levels A and C by increasing their relativities by a proportional amount

premium for levels A & C	=	135,909	+	215,303	=	351,212
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required relativity increase	=	7,988	/	351,212	=	2.274% <==== A & C relativity increase
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Step 3b BUT, we must now back out the base rate decrease from A & C so we don't "lose" any of the new premium

Level	original indicated		relativity adjustment		base rate adjustment		adjusted relativities	
A	0.65	x	1.0227	/	0.9900	=	0.6715	<==== revised Level A relativity
B	1.00						1.0000	<==== base level (no change)
C	1.05	x	1.0227	/	0.9900	=	1.0847	<==== revised Level C relativity

$$\begin{array}{c} \wedge \\ / \quad \backslash \\ | \end{array}$$

final answers

Reading: Werner 14: Implementation
Model: Text Example
Problem Type: Additive Expense Fee

Pricing-15: Experience Modification - CGL (Problem)

Find Calculate the following:
(a) fixed expense ratio
(b) fixed additive expense fee

Given

countrywide premium (\$000s)	9,000
profit provision	6%
average loss cost	220

expense category	countrywide expenses (\$000s)	%-fixed
commissions	1,400	0%
general expenses	1,200	45%
other acquisition	500	100%
taxes	200	0%
licenses & fees	50	100%
TOTAL	3,350	

Pricing-15: Experience Modification - CGL (Solution)

Step 1 calculate \$-fixed based on %-fixed

	\$-total		%-fixed		\$-fixed
commissions	1,400	x	0%	=	0
general expenses	1,200	x	45%	=	540
other acquisition	500	x	100%	=	500
taxes	200	x	0%	=	0
licenses & fees	50	x	100%	=	50
	3,350				1,090

Step 2a calculate the fixed expense ratio F

$$\begin{aligned}
 F &= \frac{\$-fixed}{\$-total} = \frac{1,090}{3,350} \\
 &= 12.1\% \quad \text{<== final answer (a)}
 \end{aligned}$$

Step 2b calculate other ratios we'll need in Step 3

$$\begin{aligned}
 V + F &= \frac{\$-total}{\$-total} = \frac{3,350}{9,000} = 37.2\% \\
 V + F + Q &= \frac{V + F}{V + F + Q} + \frac{Q}{V + F + Q} = \frac{37.2\%}{43.2\%} + \frac{6.0\%}{43.2\%} = 43.2\% \\
 V + Q &= \frac{V + F + Q}{V + F + Q} - \frac{F}{V + F + Q} = \frac{43.2\%}{43.2\%} - \frac{12.1\%}{43.2\%} = 31.1\%
 \end{aligned}$$

Step 3a calculate $\bar{P}(p)$ [projected avg prem] and $\bar{E}(F)(p)$ [projected fixed expense] as intermediate steps

$$\begin{aligned}
 \bar{P}(p) &= \frac{\text{loss cost}}{\bar{P}(p)} = \frac{220}{56.8\%} = 387.48 \\
 \bar{E}(F)(p) &= \bar{P}(p) \times F = 387.48 \times 12.1\% = 46.93
 \end{aligned}$$

Step 3b put everything together to get the final projected fixed additive expense fee A(p)

$$\begin{aligned}
 A(p) &= \frac{\bar{E}(F)(p)}{(1-V-Q)} = \frac{46.93}{68.9\%} \\
 &= 68.12 \quad \text{<== final answer (b)}
 \end{aligned}$$

Note 1: The quantity $(1-V-Q)$ is called the Variable Permissible Loss Ratio or VPLR.

Note 2: I used a "p" in parentheses (p) to indicate "projected" quantities. Strictly speaking, the "p" should be a subscript but it was too small to be legible in this spreadsheet.

Reading: Werner 14: Implementation **Pricing-15: Experience Modification - WC (Problem)**
Model: 2015.Fall #11 (without minimum premium requirement)
Problem Type: Base Rate - Extension of Exposures Method

Find Calculate the base rate required to achieve an average rate increase of 15%

Given current base rate 1,250

Relativities

AOI levels	current	indicated
less than 100,000	0.750	0.600
equal to or above 100,000	1.000	1.200

Territories	current	indicated
territory 1	0.800	0.850
territory 2	1.000	1.000

In-Force Exposures

AOI levels	Terr 1	Terr 2
less than 100,000	1,500	4,000
equal to or above 100,000	1,500	3,000

Fixed Expense Fee

	current	indicated
Fixed Expense Fee	0	0

Pricing-15: Experience Modification - WC (Solution)

Preliminary Step: rebase the indicated relativies so the base level relativity for each variable is 1.0

AOI levels	current	indicated
less than 100,000	0.750	0.500
equal to or above 100,000	1.000	1.000

* rebased

Territories	current	indicated
territory 1	0.800	0.850
territory 2	1.000	1.000

* rebased

Step 1 calculate the current average premium by rerating every combination of AOI x Territory
(we can then infer the proposed average premium)

AOI	Territory	current AOI fctr	current Terr fctr	current fixed fee	in-force exposures	current premium	
< 100K	1	0.750	0.800	0	1,500	1,125,000	= (base x AOI x terr + fee) x (in-force exposures)
>= 100K	1	1.000	0.800	0	1,500	1,500,000	
< 100K	2	0.750	1.000	0	4,000	3,750,000	
>= 100K	2	1.000	1.000	0	3,000	3,750,000	
					10,000	10,125,000	

====> current avg prem = 1,012.50

proposed average premium = 1,012.50 x 1.15 <==== apply 15% increase
= 1,164.38

Step 2 use an arbitrary base seed value B to calculate the proposed average premium by rerating every combination of AOI x Terr

AOI	Territory	proposed AOI fctr	proposed Terr fctr	proposed fixed fee	in-force exposures	proposed premium	
< 100K	1	0.500	0.850	0	1,500	637,500	= (base x AOI x terr + fee) x (in-force exposures)
>= 100K	1	1.000	0.850	0	1,500	1,275,000	
< 100K	2	0.500	1.000	0	4,000	2,000,000	
>= 100K	2	1.000	1.000	0	3,000	3,000,000	
					10,000	6,912,500	

base seed value = 1,000

====> seed avg prem = 691.25

Step 3 calculate the final proposed base rate by adjusting the base seed value appropriately

final proposed base rate
= seed x (proposed avg prem - indicated fee) / (seed avg prem - indicated fee)
= 1,000 x (1164.38 - 0) / (691.25 - 0)
= 1,684.45 <== final answer

Note: The actual exam problem assumed the fixed expense fee was 0 and also imposed a minimum premium requirement.

Reading: Werner 14: Implementation
Model: Text Example
Problem Type: AARD Method

Pricing-15: Loss-Rated Composite Rating (Problem)

Find Calculate the base rate required to achieve an average rate increase of

15%

Given current average premium

1,012.50

Relativities

AOI levels	current	indicated	expos.
less than 100,000	0.750	0.600	5,500
equal to or above 100,000	1.000	1.200	4,500

Territories	current	indicated	expos.
territory 1	0.800	0.850	3,000
territory 2	1.000	1.000	7,000

Fixed Expense Fee

	current	indicated
Fixed Expense Fee	0	0

Pricing-15: Loss-Rated Composite Rating (Solution)

Preliminary Step: rebase the indicated relativities so the base level relativity for each variable is 1.0

AOI levels	current	indicated
less than 100,000	0.750	0.500
equal to or above 100,000	1.000	1.000

** rebased*

Territories	current	indicated
territory 1	0.800	0.850
territory 2	1.000	1.000

** rebased*

Step 1 calculate the product of the exposure-weighted averages of the rebased indicated relativities: $\bar{S}(p)$

$$\begin{aligned}
 \text{AOI average relativity} &= 0.7250 \\
 \text{Territory average relativity} &= 0.9550 \\
 &\quad \underline{0.6924} \quad \text{<==== product = } \bar{S}(p)
 \end{aligned}$$

Step 2 calculate the proposed average premium: $\bar{P}(p)$

$$\begin{aligned}
 \bar{P}(p) &= (\text{current average premium}) \times (1 + \text{rate change}) \\
 &= 1,012.50 \times 1.15 \\
 &= 1,164.38
 \end{aligned}$$

Step 3 calculate the proposed base rate $B(p)$

$$\begin{aligned}
 B(p) &= \left(\bar{P}(p) - A(p) \right) / \bar{S}(p) \\
 &= \left(1,164.38 - 0 \right) / 0.6924 \\
 &= 1,681.71 \quad \text{<== final answer}
 \end{aligned}$$

Reading: Werner 14: Implementation
Model: Text Example
Problem Type: A(Δ)ARD Method

Pricing-15: Large Deductible Policies (Problem)

Find Calculate the base rate required to achieve an average rate increase of 15%
Given current base rate 1,250 current average premium 1,012.50

Relativities

AOI levels	current	indicated	expos.
less than 100,000	0.750	0.600	5,500
equal to or above 100,000	1.000	1.200	4,500

Territories	current	indicated	expos.
territory 1	0.800	0.850	3,000
territory 2	1.000	1.000	7,000

Fixed Expense Fee

	current	indicated
Fixed Expense Fee	0	0

Pricing-15: Large Deductible Policies (Solution)

Preliminary Step: rebase the indicated relativities so the base level relativity for each variable is 1.0

AOI levels	current	indicated	ind / curr
< 100,000	0.750	0.500	0.667
>= 100,000	1.000	1.000	1.000
exposure-wtd total	0.863	0.725	

Territories	current	indicated	ind / curr
territory 1	0.800	0.850	1.063
territory 2	1.000	1.000	1.000
exposure-wtd total	0.940	0.955	

Step 1 calculate the product of (total indicated) / (total current) across all rating vars: $1+\Delta s\%$

$$\begin{array}{lcl}
 \text{AOI:} & (\text{total indicated}) / (\text{total current}) & = 0.725 / 0.863 = 0.841 \\
 \text{Territory:} & (\text{total indicated}) / (\text{total current}) & = 0.955 / 0.940 = 1.016 \\
 & & \underline{0.854} = 1+\Delta s\%
 \end{array}$$

Step 2 calculate the proposed average premium: $\bar{P}(p)$

$$\begin{array}{lcl}
 \bar{P}(p) & = & (\text{current average premium}) \times (1 + \text{rate change}) \\
 & = & 1,012.50 \times 1.15 \\
 & = & 1,164.38
 \end{array}$$

Step 3a calculate the proposed base rate adjustment

$$\begin{array}{lcl}
 \text{adjustment} & = & [\bar{P}(p) - A(p)] / [\bar{P}(c) - A(c)] \times 1 / (1+\Delta s\%) \\
 & = & 1.150 \times 1.171 \\
 & = & 1.347
 \end{array}$$

Step 3b calculate the proposed base rate $B(p)$

$$\begin{array}{lcl}
 B(p) & = & B(c) \times \text{adjustment} \\
 & = & 1,250 \times 1.347 \\
 & = & 1,683.27
 \end{array}$$

Reading: Werner 14: Implementation
Model: Text Example
Problem Type: A(Δ)ARD Method

Pricing-15: Retrospective Rating (Problem)

Find Calculate the base rate required to achieve an average rate increase of 15%

Given current base rate 1,250 current average premium 1,012.50

Relativities

AOI levels	current	indicated	expos.
less than 100,000	0.750	0.600	5,500
equal to or above 100,000	1.000	1.200	4,500

Territories	current	indicated	expos.
territory 1	0.800	0.850	3,000
territory 2	1.000	1.000	7,000

Fixed Expense Fee

	current	indicated
Fixed Expense Fee	0	0

Pricing-15: Retrospective Rating (Solution)

Preliminary Step: rebase the indicated relativies so the base level relativity for each variable is 1.0

AOI levels	current	indicated	ind / curr
< 100,000	0.750	0.500	0.667
>= 100,000	1.000	1.000	1.000
exposure-wtd total	0.863	0.725	

Territories	current	indicated	ind / curr
territory 1	0.800	0.850	1.063
territory 2	1.000	1.000	1.000
exposure-wtd total	0.940	0.955	

Step 1 calculate the product of (total indicated) / (total current) across all rating vars: $1+\Delta s\%$

$$\begin{array}{lcl}
 \text{AOI:} & (\text{total indicated}) / (\text{total current}) & = 0.725 / 0.863 = 0.841 \\
 \text{Territory:} & (\text{total indicated}) / (\text{total current}) & = 0.955 / 0.940 = 1.016 \\
 & & \underline{0.854} = 1+\Delta s\%
 \end{array}$$

Step 2 calculate the proposed average premium: $\bar{P}(p)$

$$\begin{array}{lcl}
 \bar{P}(p) & = & (\text{current average premium}) \times (1 + \text{rate change}) \\
 & = & 1,012.50 \times 1.15 \\
 & = & 1,164.38
 \end{array}$$

Step 3a calculate the proposed base rate adjustment

$$\begin{array}{lcl}
 \text{adjustment} & = & [\bar{P}(p) - A(p)] / [\bar{P}(c) - A(c)] \times 1 / (1+\Delta s\%) \\
 & = & 1.150 \times 1.171 \\
 & = & 1.347
 \end{array}$$

Step 3b calculate the proposed base rate $B(p)$

$$\begin{array}{lcl}
 B(p) & = & B(c) \times \text{adjustment} \\
 & = & 1,250 \times 1.347 \\
 & = & 1,683.27
 \end{array}$$

Reading: Werner Appendix B: Homeowners
Model: Text Example
Problem Type: Pure Premium Rate Indication for Homeowners

Pricing-Appendix: HO Indication (Problem)

Find Calculate the indicated rate given the following information:

Given

effective date	2026	4	1
	year	month	day

term	12	1
	months	ren

V: variable expense	20.0%
Q: profit provision	7.0%
E(F): fixed expenses	\$ 65.00 <-- through 2024

non-modeled cat-to-AIY ratio	0.350
2024 reinsurance cost	66,000
2024 reinsurance recoveries	30,000

* assume the net per-exposure cost of reinsurance
for the effective period is the same as for CY 2024

historical loss trend	4.0%
projected loss trend	2.0%
fixed expense trend	3.0%
exposure trend	0.0%

* modeled cat pure premium	34.31
	* projected

CY	EE	non-cat rpt loss & ALAE	LDF	ULAE factor	AIY / EE exp fit
2020	730	215,000	1.00	1.050	256
2021	930	288,600	1.06	1.050	270
2022	990	336,400	1.10	1.050	284
2023	1,040	354,700	1.14	1.050	299
2024	1,090	435,700	1.20	1.050	315
2025					332
2026					350
2027					369

Pricing-Appendix: HO Indication (Solution)

Step 1	projected.....	ultimate.....	non-cat.....	pure premium	=	444.14	
Step 2a	projected.....	non-modeled...	cat.....	pure premium	=	133.86	
Step 2b	projected.....	modeled.....	cat.....	pure premium	=	34.31	(given)
Step 3	projected.....	net.....	reins.....	pure premium	=	33.03	
Step 4	projected.....	fixed.....	expense.....	pure premium	=	69.47	
						total =====>	714.81
						divide by VPLR =====>	979.19 <== final answer
						VPLR = 73.0%	

Step 1 calculate projected ultimate non-cat pure premium

	trend periods		non-cat ultimate LOSS	non-cat ultimate Pure Pr.	
	from CY to 2024	from 2024 to eff. per.			
2020	4	2.75	278,876	382.02	
2021	3	2.75	381,542	410.26	
2022	2	2.75	443,767	448.25	
2023	1	2.75	466,272	448.34	
2024	0	2.75	579,707	531.84	
				average of column ==>	444.14

non-cat ultimate LOSS

= (non-cat rptd loss & ALAE) x LDF x ULAE x trends

= 215000 x 1 x 1.05 x (1.04)^4 x (1.02)^2.75

= 288600 x 1.06 x 1.05 x (1.04)^3 x (1.02)^2.75

= 336400 x 1.1 x 1.05 x (1.04)^2 x (1.02)^2.75

= 354700 x 1.14 x 1.05 x (1.04)^1 x (1.02)^2.75

= 435700 x 1.2 x 1.05 x (1.04)^0 x (1.02)^2.75

Step 2a calculate projected non-modeled cat pure premium

$$\begin{aligned}
 &= \text{cat-to-AIY-ratio} \quad \times \quad \text{AIY-to-EE} \quad \times \quad \text{ULAE factor} \\
 &= 0.350 \quad \times \quad 364.25 \quad \times \quad 1.050 \\
 &= 133.86 \quad \text{<== final answer for Step 2a}
 \end{aligned}$$

side calc to get appropriate value for AIY-to-EE:

The AAD (Average Accident Date) for the AIY-to-EE value must line up with the AAD of the effective period.

AIY-to-EE is a weighted average of AIY-to-EE values for CY 2026 and CY 2027 where the weight given to CY 2026 is:

AAD(eff. per.)	=	2027 - 04 - 01
AAD(2026)	=	2026 - 07 - 01
AAD(2027)	=	2027 - 07 - 01

0.25

$$\text{AIY-to-EE} = 0.25 \times 350 + 0.75 \times 369 = 364.25$$

Step 3 calculate the net reinsurance cost per exposure (assume no exposure trend so use exposures from 2024)

$$\begin{aligned}
 \text{net cost} &= (2024 \text{ reinsurance cost} - 2024 \text{ reinsurance recoveries}) / \text{EE for CY 2024} \\
 &= (66,000 - 30,000) / 1,090 \\
 &= 33.03 \quad \text{<== final answer for Step 3}
 \end{aligned}$$

Step 4 trend fixed expenses using AWD (since most fixed expenses are incurred when policy is written)

$$\begin{aligned}
 \text{AWD(2024)} &= 2024 - 07 - 01 \\
 \text{AWD(Eff. Per.)} &= 2026 - 10 - 01
 \end{aligned}$$

$$\text{trend period} = 2.25$$

projected fixed expense pure premium

$$\begin{aligned}
 &= \text{E(F) through 2024} \quad \times \quad (1 + \text{trend})^{(\text{trend period})} \\
 &= 65.00 \quad \times \quad (1.03)^{2.25} \\
 &= 69.47 \quad \text{<== final answer for Step 4}
 \end{aligned}$$

Reading: Werner Appendix D: Worker's Compensation
Model: Text Example
Problem Type: Loss Ratio Rate Indication for WC

Pricing-Appendix: WC Indication (Problem)

Find Calculate the final company rate change using both industry and company data.

Given

information required for step 1 of solution: PROJECTED LOSS COST PREMIUM

effective date: 2023 1 1 (year, month, day)
 rates in effect for 12 months
 policy term: 12 months

AY	industry loss cost premium	annual payroll change	Historical Experience Mod (HEM)
2020	2,770	-0.5%	0.960
2021	3,150	5.0%	0.960
2022	2,610	4.0%	0.860

* loss cost premium is already at CRL (Current Rate Level)

Projected Annual Wage Change (PAWC) 1.0%
 Expected Experience Modification (EEM) 0.930

information required for step 2 of solution: PROJECTED MEDICAL LOSS RATIO

projected medical fee schedule change: 1.0% = fee % change
 projected other medical change: 3.0% = other % change

portion of medical loss subject to fee schedule = m 80.0% use this fee % to calculate a weighted average

AY	Rptd Med Loss	Med Loss LDF to Ult	Med Fee Sched Change	Other Medical Change
2020	1,243	1.000	-13.0%	2.0%
2021	1,411	1.800	-2.0%	1.0%
2022	1,198	2.000	12.0%	2.0%

information required for step 3 of solution: INDUSTRY & COMPANY INDICATED RATE CHANGES

Step 3a indemnity cost loss ratio: 18.0%
 LAE ratio to ult loss: 22.0%

Step 3b V + Q: 28.0%
 expected loss cost difference: 4.0%
 current deviation: 1.880

Here are some notes on STEP 2 of the solution that didn't fit on the solution page:

- (5) = weighted average of (3) and (4) with weights m and (1-m)
- (6) = product of (1.0 + "lower" entries) from (5)
- (7) = $m \times (1.0 + \text{med \% change})^{\text{trend period}} + (1 - m) \times (1.0 + \text{other \% change})^{\text{trend period}}$
 $= 0.8 \times (1.01)^{1.5} + 0.2 \times (1.03)^{1.5}$
- (8) = (1) x (2) x (6) x (7)
- (9) = (8) / (projected loss cost premium from Step 1b)

Pricing-Appendix: WC Indication (Solution)

Step 1 calculate the projected loss cost premium (WC advisory loss costs)

1a trend period for 'step 2' in '2-step' trending

=	(AAD for latest available year)	to	(AAD of effective period)
=	2022 7 1	to	2024 1 1
=	1.5000 years		

trend factor = $(1 + \text{PAWC} \cdot 1.0\%)^{1.5} = 1.0150$

1b calculate the 'projected' loss cost premium'

	(1) (this is the given information)	(2)	(3)	(4)	(5)	(6)	(7)	
	industry loss cost premium	annual payroll change	Hist. Exp. Mod (HEM)	factor to current wage level	expected future wage level change	experience mod factor	projected loss cost premium	
2020	2,770	-0.5%	0.960	1.0920	1.0150	0.9688	2,974.38	<==== final answers to step 1
2021	3,150	5.0%	0.960	1.0400	1.0150	0.9688	3,221.35	<==== final answers to step 1
2022	2,610	4.0%	0.860	1.0000	1.0150	1.0814	2,864.88	<==== final answers to step 1
							9,060.61	<==== final answers to step 1

- (4) = $(1.0 + (2)\text{NextRow}) \times (4\text{NextRow})$ = product of $(1.0 + \text{"lower" entries})$ from (2)
 (5) = $(1 + \text{PAWC})^{(\text{trend period})}$ <==== trend factor from step 1a
 (6) = $\text{EEM} / (3) = \text{EEM} / \text{HEM}$ <==== this is like 'on-leveling' the experience modification
 (7) = $(1) \times (4) \times (5) \times (6)$

Notes: - column (4) is similar to 'step 1' in '2-step' trending
 - column (5) is similar to 'step 2' in '2-step' trending
 - column (6) is similar on-leveling premium except here we're 'on-leveling' the experience modification

Step 2 calculate the projected medical loss ratio

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
	Rptd Med Loss	Med Loss LDF to Ult	Med Fee Sched Change	Other Medical Change	combined effect of medical trends	factor to current med cost level	combined effect of projected trend	projected ultimate medical loss	projected ultimate medical LR	
2020	1,243	1.000	-13.0%	2.0%	-10.00%	1.085	1.021	1,376.60	46.28%	
2021	1,411	1.800	-2.0%	1.0%	-1.40%	1.100	1.021	2,852.72	88.56%	
2022	1,198	2.000	12.0%	2.0%	10.00%	1.000	1.021	2,446.55	85.40%	
								6,675.87	73.68%	<==== final ans to step 2

Step 3 calculate the industry and company rate changes

industry indicated rate change

= $(\text{med LR} + \text{indem LR}) \times (1 + \text{LAE ratio}) - 1.0$
 = $(73.7\% + 18.0\%) \times (1 + 22\%) - 1.0$
 = 11.85% <==== industry rate change (assumes V+Q = 0)

proposed deviation from industry

= (expense & profit adjustment) x (operational adjustment)
 = $1 / (1 - V - Q) \times (1 + \text{expected loss cost difference})$
 = 1.4444

company indicated rate change

= (proposed deviation) / (current deviation) x $(1 + \text{industry chg}) - 1.0$
 = -14.06% <==== FINAL ANSWER!!