



PowerPack Station 2: One-Stop PDF

RESERVING

## TABLE OF CONTENTS

|             |                            |    |
|-------------|----------------------------|----|
| Reserving A | Information in Triangles   | 3  |
|             | Building AY Triangles      | 5  |
|             | Development Method         | 7  |
|             | BF Method                  | 9  |
|             | CC Method                  | 11 |
| Reserving B | FS Method                  | 13 |
|             | Case OS Method 2 - Formula | 15 |
|             | RBS Method                 | 17 |
| Reserving C | Building RY Triangles      | 19 |
|             | Tail Factors               | 21 |
|             | BF Method - Tort Reform    | 23 |
|             | CC Method - Trends         | 25 |
| Reserving D | FS Method - Disposal Rates | 27 |
|             | Case OS Method 1           | 29 |
|             | PBS Method                 | 31 |
|             | Emergence                  | 33 |
| Reserving E | SS Ratio Method            | 35 |
|             | Reinsurance                | 37 |
|             | ALAE - Multiplicative      | 39 |
|             | ALAE - Additive            | 41 |
|             | ULAE - Classical           | 43 |
|             | ULAE - Kittel              | 45 |
|             | ULAE - Conger-Nolibos      | 47 |

**Reading:** Friedland 05 (Triangles)  
**Model:** Supplementary Problems based on Video  
**Problem Type:** Information in Triangles

## Reserving-A: Information in Triangles (Problem)

|      |     |                                  |      |   |    |        |
|------|-----|----------------------------------|------|---|----|--------|
| Find | (a) | cumulative paid loss for AY      | 2024 | @ | 24 | months |
|      | (b) | incremental paid loss for AY     | 2023 | @ | 36 | months |
|      | (c) | cumulative reported loss for AY  | 2023 | @ | 24 | months |
|      | (d) | incremental reported loss for AY | 2022 | @ | 24 | months |
|      | (e) | cumulative reported count for AY | 2023 | @ | 36 | months |
|      | (f) | total paid loss for CY           | 2025 |   |    |        |
|      | (g) | total rptd loss for CY           | 2025 |   |    |        |

Given

| Amounts | Paid Loss | Development Age |       |       |        |        |
|---------|-----------|-----------------|-------|-------|--------|--------|
| AY      | 12        | 24              | 36    | 48    | 60     | 72     |
| 2020    | 2,160     | 5,040           | 7,200 | 9,360 | 10,080 | 11,520 |
| 2021    | 1,217     | 3,650           | 6,084 | 7,301 | 7,909  |        |
| 2022    | 1,080     | 2,700           | 4,320 | 5,400 |        |        |
| 2023    | 469       | 1,406           | 2,343 |       |        |        |
| 2024    | 399       | 1,197           |       |       |        |        |
| 2025    | 950       |                 |       |       |        |        |

| Amounts | O/S Loss | Development Age |       |       |       |    |
|---------|----------|-----------------|-------|-------|-------|----|
| AY      | 12       | 24              | 36    | 48    | 60    | 72 |
| 2020    | 3,600    | 2,880           | 2,160 | 1,440 | 1,440 | 0  |
| 2021    | 3,650    | 3,042           | 1,825 | 1,217 | 1,217 |    |
| 2022    | 2,160    | 1,620           | 1,080 | 540   |       |    |
| 2023    | 1,406    | 1,406           | 937   |       |       |    |
| 2024    | 1,197    | 1,197           |       |       |       |    |
| 2025    | 950      |                 |       |       |       |    |

<== same as Case O/S

| Amounts | Paid Count | Development Age |    |    |    |    |
|---------|------------|-----------------|----|----|----|----|
| AY      | 12         | 24              | 36 | 48 | 60 | 72 |
| 2020    | 3          | 7               | 10 | 13 | 14 | 16 |
| 2021    | 2          | 6               | 10 | 12 | 13 |    |
| 2022    | 2          | 5               | 8  | 10 |    |    |
| 2023    | 1          | 3               | 5  |    |    |    |
| 2024    | 1          | 3               |    |    |    |    |
| 2025    | 2          |                 |    |    |    |    |

| Amounts | O/S Count | Development Age |    |    |    |    |
|---------|-----------|-----------------|----|----|----|----|
| AY      | 12        | 24              | 36 | 48 | 60 | 72 |
| 2020    | 5         | 4               | 3  | 2  | 2  | 0  |
| 2021    | 6         | 5               | 3  | 2  | 2  |    |
| 2022    | 4         | 3               | 2  | 1  |    |    |
| 2023    | 3         | 3               | 2  |    |    |    |
| 2024    | 3         | 3               |    |    |    |    |
| 2025    | 2         |                 |    |    |    |    |

10   4   11   8   8   8   8   8   8   8   8   6

## Reserving-A: Information in Triangles (Solution)

- (a) 469 <== directly from paid loss triangle
- (b) 469 = 469 - 0
- (c) 9,360 = 7,200 + 2,160
- (d) 1,440 = 7,200 + 2,160 - 5,040 - 2,880
- (e) 14 = 12 + 2
- (f) 4,893 <== directly from alternate format of paid loss triangle below
- (g) 4,942 <== directly from alternate format of reported loss triangle below

side calc for CY loss: express triangles in alternate format

| Amounts | Paid Loss |       | as of Calendar Year End |        |        |        |
|---------|-----------|-------|-------------------------|--------|--------|--------|
| AY      | 2020      | 2021  | 2022                    | 2023   | 2024   | 2025   |
| 2020    | 2,160     | 5,040 | 7,200                   | 9,360  | 10,080 | 11,520 |
| 2021    |           | 1,217 | 3,650                   | 6,084  | 7,301  | 7,909  |
| 2022    |           |       | 1,080                   | 2,700  | 4,320  | 5,400  |
| 2023    |           |       |                         | 469    | 1,406  | 2,343  |
| 2024    |           |       |                         |        | 399    | 1,197  |
| 2025    |           |       |                         |        |        | 950    |
|         | 2,160     | 6,257 | 11,930                  | 18,613 | 23,506 | 29,320 |
| CY paid | 2,160     | 4,097 | 5,674                   | 6,682  | 4,893  | 5,814  |
| CY      | 2020      | 2021  | 2022                    | 2023   | 2024   | 2025   |

| Amounts | Rptd Loss |        | as of Calendar Year End |        |        |        |
|---------|-----------|--------|-------------------------|--------|--------|--------|
| AY      | 2020      | 2021   | 2022                    | 2023   | 2024   | 2025   |
| 2020    | 5,760     | 7,920  | 9,360                   | 10,800 | 11,520 | 11,520 |
| 2021    |           | 4,867  | 6,692                   | 7,909  | 8,518  | 9,126  |
| 2022    |           |        | 3,240                   | 4,320  | 5,400  | 5,940  |
| 2023    |           |        |                         | 1,874  | 2,812  | 3,280  |
| 2024    |           |        |                         |        | 1,596  | 2,394  |
| 2025    |           |        |                         |        |        | 1,901  |
|         | 5,760     | 12,787 | 19,292                  | 24,904 | 29,845 | 34,161 |
| CY rptd | 5,760     | 7,027  | 6,505                   | 5,611  | 4,942  | 4,316  |
| CY      | 2020      | 2021   | 2022                    | 2023   | 2024   | 2025   |



**Reading:** Friedland 05 (Triangles)  
**Model:** 2014.Fall #14  
**Problem Type:** Building Triangles from Raw Claims Data

## Reserving-A: Building Triangles (Problem)

**Find** Build the **AY (Accident Year)** paid & reported claim triangles. (Unless stated otherwise these are cumulative triangles)

**Given**

| claim id | AY   | RY   | <u>2020</u> |          | <u>2021</u> |          | <u>2022</u> |          |
|----------|------|------|-------------|----------|-------------|----------|-------------|----------|
|          |      |      | paid in     | ending   | paid in     | ending   | paid in     | ending   |
|          |      |      | CY          | case O/S | CY          | case O/S | CY          | case O/S |
| 1        | 2020 | 2020 | 600         | 0        | 0           | 0        | 0           | 0        |
| 2        | 2020 | 2020 | 700         | 0        | 0           | 0        | 0           | 0        |
| 3        | 2020 | 2021 | -           | -        | 900         | 300      | 400         | 0        |
| 4        | 2020 | 2022 | -           | -        | -           | -        | 600         | 0        |
| 5        | 2020 | 2022 | -           | -        | -           | -        | 600         | 100      |
| 6        | 2021 | 2021 | -           | -        | 300         | 0        | 0           | 0        |
| 7        | 2021 | 2021 | -           | -        | 500         | 400      | 500         | 200      |
| 8        | 2021 | 2021 | -           | -        | 800         | 400      | 400         | 200      |
| 9        | 2021 | 2021 | -           | -        | 600         | 200      | 400         | 200      |
| 10       | 2022 | 2022 | -           | -        | -           | -        | 700         | 100      |
| 11       | 2022 | 2023 | -           | -        | -           | -        | -           | -        |
| 12       | 2022 | 2023 | -           | -        | -           | -        | -           | -        |
| 13       | 2022 | 2024 | -           | -        | -           | -        | -           | -        |
| 14       | 2022 | 2024 | -           | -        | -           | -        | -           | -        |
| 15       | -    | -    | -           | -        | -           | -        | -           | -        |
| 16       | -    | -    | -           | -        | -           | -        | -           | -        |
| 17       | -    | -    | -           | -        | -           | -        | -           | -        |
| 18       | -    | -    | -           | -        | -           | -        | -           | -        |

## Reserving-A: Building Triangles (Solution)

**Step 1** Sum across **AY** (This is simple in Excel but easy to make a mistake with pencil and paper.)

**Suggestion:** Use a ruler to draw a horizontal line between one AY and the next.

| AY   | RY  | <u>2020</u>   |                          | <u>2021</u>   |                          | <u>2022</u>   |                          |
|------|-----|---------------|--------------------------|---------------|--------------------------|---------------|--------------------------|
|      |     | paid in<br>CY | ending<br>case O/S<br>CY | paid in<br>CY | ending<br>case O/S<br>CY | paid in<br>CY | ending<br>case O/S<br>CY |
| 2020 | all | 1,300         | 0                        | 900           | 300                      | 1,600         | 100                      |
| 2021 | all | 0             | 0                        | 2,200         | 1,000                    | 1,300         | 600                      |
| 2022 | all | 0             | 0                        | 0             | 0                        | 700           | 100                      |
| 2023 | all | 0             | 0                        | 0             | 0                        | 0             | 0                        |

**Step 2** Transfer the colour-coded values from Step 1 into these triangles...

| incremental PAID claims |       |       |       | case O/S |       |     |     |
|-------------------------|-------|-------|-------|----------|-------|-----|-----|
| AY                      | 12    | 24    | 36    | AY       | 12    | 24  | 36  |
| 2020                    | 1,300 | 900   | 1,600 | 2020     | 0     | 300 | 100 |
| 2021                    | 2,200 | 1,300 |       | 2021     | 1,000 | 600 |     |
| 2022                    | 700   |       |       | 2022     | 100   |     |     |
| 2023                    |       |       |       | 2023     |       |     |     |

**Step 3** Form the CUMULATIVE paid triangle by summing across rows in the incremental paid triangle

(final  
answer)

| cumulative PAID claims |       |       |       |
|------------------------|-------|-------|-------|
| AY                     | 12    | 24    | 36    |
| 2020                   | 1,300 | 2,200 | 3,800 |
| 2021                   | 2,200 | 3,500 |       |
| 2022                   | 700   |       |       |
| 2023                   |       |       |       |

Ex:  $2,200 = 1,300 + 900$   
 $3,800 = 1,300 + 900 + 1,600$

**Step 4** Form the CUMULATIVE reported triangle:  $\text{cumulative reported claims} = (\text{cumulative paid claims}) + (\text{Case O/S})$

(final  
answer)

| cumulative REPORTED claims |       |       |       |
|----------------------------|-------|-------|-------|
| AY                         | 12    | 24    | 36    |
| 2020                       | 1,300 | 2,500 | 3,900 |
| 2021                       | 3,200 | 4,100 |       |
| 2022                       | 800   |       |       |
| 2023                       |       |       |       |

**Reading:** Friedland 07 (Development Method)  
**Model:** Simple Example of Development Technique  
**Problem Type:** Paid Claims Development

## Reserving-A: Development Method (Problem)

**Find** Calculate ultimate claims for all accident years using data as of year-end

2023

**Given**

**cumulative paid claims**

| AY   | 12    | 24    | 36    | 48    |
|------|-------|-------|-------|-------|
| 2020 | 1,590 | 1,591 | 1,655 | 1,671 |
| 2021 | 1,606 | 1,792 | 1,840 |       |
| 2022 | 1,605 | 1,700 |       |       |
| 2023 | 1,604 |       |       |       |

Assume no development past 48 months. In other words, the triangle is fully developed by 48 months.

The paid claims on the latest diagonal are in **brown font** for instructional purposes within the solution.

## Reserving-A: Development Method (Solution)

|                   |       |                                                       |                                                                            |        |        |        |
|-------------------|-------|-------------------------------------------------------|----------------------------------------------------------------------------|--------|--------|--------|
| Step A            | ====> | link ratios for paid claim triangle                   |                                                                            |        |        |        |
|                   |       | AY                                                    | 12-24                                                                      | 24-36  | 36-48  | 48-ult |
|                   |       | 2020                                                  | 1.001                                                                      | 1.040  | 1.010  |        |
|                   |       | 2021                                                  | 1.116                                                                      | 1.027  |        |        |
|                   |       | 2022                                                  | 1.059                                                                      |        |        |        |
|                   |       | 2023                                                  |                                                                            |        |        |        |
| Step B            | ====> | selected                                              | 1.059                                                                      | 1.034  | 1.010  | 1.050  |
| Step C            | ====> | calculate age-to-ultimate LDFs                        |                                                                            |        |        |        |
|                   |       |                                                       | 12-ult                                                                     | 24-ult | 36-ult | 48-ult |
|                   |       | age -> ult                                            | 1.160                                                                      | 1.096  | 1.060  | 1.050  |
|                   |       |                                                       | <===== (selected) x (prior [age -> ult])<br>(calculate from right-to-left) |        |        |        |
| Step D            | ====> | calculate ultimate losses based on latest paid losses |                                                                            |        |        |        |
|                   |       |                                                       | '23@12                                                                     | '22@24 | '21@36 | '20@48 |
|                   |       | diagonal                                              | 1,604                                                                      | 1,700  | 1,840  | 1,671  |
| final answers ==> |       | ultimate                                              | 1,860                                                                      | 1,863  | 1,951  | 1,755  |
|                   |       |                                                       | <===== (diagonal) x (age -> ult)                                           |        |        |        |

Sometimes it's nice to present the ultimates in a column to the right of the original triangle:

| cumulative paid claims |       |       |       |       | estimated | real      |         |
|------------------------|-------|-------|-------|-------|-----------|-----------|---------|
| AY                     | 12    | 24    | 36    | 48    | ultimates | ultimates | % error |
| 2020                   | 1,590 | 1,591 | 1,655 | 1,671 | 1,755     | 240       | 631%    |
| 2021                   | 1,606 | 1,792 | 1,840 |       | 1,951     | 240       | 713%    |
| 2022                   | 1,605 | 1,700 |       |       | 1,863     | 240       | 676%    |
| 2023                   | 1,604 |       |       |       | 1,860     | 240       | 675%    |

### Interesting side note:

This example was created using my simulation software [SimPolicy](#). One of the input parameters to the simulation is the value of the ultimate loss. For this simulation, each AY was given the same **ultimate loss of 240**. That means we can see how accurate our estimates are. More to the point, we can often see how inaccurate our estimates are regardless of how we select our LDFs (Loss Development Factors) in Step B.

### Moral:

Don't agonize for too long over selecting LDFs. In a real-life situation there will be a lot of noise or random variation that cannot be accounted for in any reserving method. Do the best you can with the information you've got but make allowances for the fact that your estimates will never be exactly right, especially for AYs at early stages of development.

**Reading:** Friedland 09 (Bornhuetter-Ferguson Method)  
**Model:** BF Method  
**Problem Type:** Reserving Methods - Simple Example of BF

## Reserving-A: BF Method (Problem)

**Find** Calculate the ultimate for AY 2025 using the Bornhuetter-Ferguson method.

**Given** All data is as of Dec 31, 2025

|            |      |       |
|------------|------|-------|
| EP for CY  | 2024 | 1,400 |
| ECR for AY | 2024 | 78%   |
| EP for CY  | 2025 | 1,300 |
| ECR for AY | 2025 | 83%   |

|                         |       |
|-------------------------|-------|
| paid loss @ 12 mths     | 590   |
| reported loss @ 12 mths | 810   |
| reported CDF for 12-ult | 1.90  |
| paid loss @ 24 mths     | 980   |
| reported loss @ 24 mths | 1,300 |
| reported CDF for 24-ult | 1.60  |

## Reserving-A: BF Method (Solution)

|        |              |   |              |   |       |
|--------|--------------|---|--------------|---|-------|
| Step 1 | ECR ultimate | = | ECR          | x | EP    |
|        |              | = | 83%          | x | 1,300 |
|        |              | = | <u>1,079</u> |   |       |

|         |                     |      |                           |
|---------|---------------------|------|---------------------------|
| Step 2a | The CDF we need is: | 1.90 | (reported CDF for 12-ult) |
|---------|---------------------|------|---------------------------|

|         |                      |                     |   |             |   |              |
|---------|----------------------|---------------------|---|-------------|---|--------------|
| Step 2b | reported BF ultimate |                     |   |             |   |              |
|         | =                    | rptd loss @ 12 mths | + | (1 - 1/CDF) | x | ECR ultimate |
|         | =                    | 810                 | + | 0.4737      | x | 1,079        |
|         | =                    | <u>1,321</u>        |   |             |   |              |

|              |                      |   |              |
|--------------|----------------------|---|--------------|
| Final Answer | ultimate for AY 2025 | = | <u>1,321</u> |
|--------------|----------------------|---|--------------|

Recall:

|        |   |          |   |          |
|--------|---|----------|---|----------|
| unpaid | = | ultimate | - | paid     |
| IBNR   | = | ultimate | - | reported |

**Reading:** Friedland 10 (Cape Cod Method)  
**Model:** 2018.Spring #18  
**Problem Type:** Reserving Methods - CC with Pure Premium Trend Adjustments

## Reserving-A: CC Method (Problem)

**Find** Calculate the **ultimate** for AY **2024** using the reported Cape Cod method.

**Given**

| AY   | rptd clms | CDF  | pure<br>premium<br>trend |
|------|-----------|------|--------------------------|
| 2023 | 8,000     | 2.50 | 1.082                    |
| 2024 | 4,900     | 3.20 | 1.040                    |
| 2025 | 2,500     | 5.60 | 1.000                    |

| CY   | EP     |
|------|--------|
| 2023 | 24,000 |
| 2024 | 22,540 |
| 2025 | 21,250 |

*\* no rate level adjustments*  
*\* no premium trend*

## Reserving-A: CC Method (Solution)

**Step 1** calculate the adjusted reported losses (*do not apply CDFs to the losses - we are not calculating the ultimate here*)

| AY   | rptd clms | x | PP trend | = | adjusted reported claims |
|------|-----------|---|----------|---|--------------------------|
| 2023 | 7,600     | x | 1.049    | = | 7,969                    |
| 2024 | 4,600     | x | 1.024    | = | 4,710                    |
| 2025 | 2,300     | x | 1.000    | = | 2,300                    |
|      |           |   |          |   | <b>14,980</b>            |

**Step 2** calculate the used-up-premium (UUP)

| CY/AY | EP     | / | CDF  | = | UUP           |
|-------|--------|---|------|---|---------------|
| 2023  | 20,520 | / | 2.30 | = | 8,922         |
| 2024  | 21,160 | / | 4.50 | = | 4,702         |
| 2025  | 17,710 | / | 5.80 | = | 3,053         |
|       |        |   |      |   | <b>16,677</b> |

**Step 3** calculate the adjusted ECR at the level: 2023 (*you must back out the pure premium trend*)

$$\begin{aligned}
 \text{ECR} &= \text{Step 1} / \text{Step 2} / (\text{PP trend for AY 2023}) \\
 &= 14,980 / 16,677 / 1.049 \\
 &= 85.66\%
 \end{aligned}$$

**Step 4** calculate the ultimate using the data for: 2023

$$\begin{aligned}
 \text{ultimate} &= \text{rptd clms} + (1 - 1 / \text{CDF}) \times \text{ECR} \times \text{EP} \\
 &= 7,600 + (1 - 1 / 2.3) \times 85.66\% \times 20,520 \\
 &= 17,535 \quad \text{<== final answer}
 \end{aligned}$$



**Reading:** Friedland 11 (Frequency-Severity Methods)  
**Model:** 2019.Spring #15  
**Problem Type:** Reserving Methods - FS (Basic Method)

## Reserving-B: FS Method (Problem)

**Find** Use a frequency-severity method to estimate the **unpaid** claims for AY 2025

**Given** paid loss @ Dec 31, 2025

700

|      | Cumulative Reported <u>Counts</u> (CRC) |     |     |     |
|------|-----------------------------------------|-----|-----|-----|
|      | 12                                      | 24  | 36  | 48  |
| 2022 | 250                                     | 238 | 245 | 260 |
| 2023 | 275                                     | 270 | 278 |     |
| 2024 | 323                                     | 320 |     |     |
| 2025 | 375                                     |     |     |     |

| AY   | Cumulative Reported <u>Loss</u> (\$000s) (CRL) |       |       |       |
|------|------------------------------------------------|-------|-------|-------|
|      | 12                                             | 24    | 36    | 48    |
| 2022 | 1,250                                          | 1,280 | 1,325 | 1,430 |
| 2023 | 1,365                                          | 1,395 | 1,450 |       |
| 2024 | 1,625                                          | 1,675 |       |       |
| 2025 | 1,900                                          |       |       |       |

\* no development past 48 months

## Step 1 develop reported counts to ultimate

====&gt; link ratios for reported count triangle

| AY              | 12-24        | 24-36        | 36-48        | 48-ult       |
|-----------------|--------------|--------------|--------------|--------------|
| 2022            | 0.952        | 1.029        | 1.061        |              |
| 2023            | 0.982        | 1.030        |              |              |
| 2024            | 0.991        |              |              |              |
| 2025            |              |              |              |              |
| <b>selected</b> | <b>0.975</b> | <b>1.030</b> | <b>1.061</b> | <b>1.000</b> |

**Tail Factor:** The triangle is fully developed as of 48 months. That means the 48-ult tail factor is equal to 1.0

====&gt; calculate age-to-ultimate LDFs

|            | 12-ult | 24-ult | 36-ult | 48-ult |
|------------|--------|--------|--------|--------|
| age -> ult | 1.065  | 1.093  | 1.061  | 1.000  |

<===== (selected) x (prior [age -> ult])  
(calculate from right-to-left)

====&gt; calculate ultimate counts based on latest reported counts

|          | '25@12 | '24@24 | '23@36 | '22@48 |
|----------|--------|--------|--------|--------|
| diagonal | 375    | 320    | 278    | 260    |
| ultimate | 399    | 350    | 295    | 260    |

<===== (diagonal) x (age -> ult)

## Step 2a calculate CRS triangle

(CRS = Cumulative Reported Severity)

| AY   | Cumulative Reported Severity (CRS) |       |       |       |
|------|------------------------------------|-------|-------|-------|
|      | 12                                 | 24    | 36    | 48    |
| 2022 | 5,000                              | 5,378 | 5,408 | 5,500 |
| 2023 | 4,964                              | 5,167 | 5,216 |       |
| 2024 | 5,031                              | 5,234 |       |       |
| 2025 | 5,067                              |       |       |       |

## Step 2b develop reported severities to ultimate

====&gt; link ratios for reported severity triangle

| AY              | 12-24        | 24-36        | 36-48        | 48-ult       |
|-----------------|--------------|--------------|--------------|--------------|
| 2022            | 1.076        | 1.006        | 1.017        |              |
| 2023            | 1.041        | 1.010        |              |              |
| 2024            | 1.040        |              |              |              |
| 2025            |              |              |              |              |
| <b>selected</b> | <b>1.052</b> | <b>1.008</b> | <b>1.017</b> | <b>1.000</b> |

**Tail Factor:** The triangle is fully developed as of 48 months. That means the 48-ult tail factor is equal to 1.0

====&gt; calculate age-to-ultimate LDFs

|            | 12-ult | 24-ult | 36-ult | 48-ult |
|------------|--------|--------|--------|--------|
| age -> ult | 1.078  | 1.025  | 1.017  | 1.000  |

<===== (selected) x (prior [age -> ult])  
(calculate from right-to-left)

====&gt; calculate ultimate severities based on latest reported severities

|          | '25@12 | '24@24 | '23@36 | '22@48 |
|----------|--------|--------|--------|--------|
| diagonal | 5,067  | 5,234  | 5,216  | 5,500  |
| ultimate | 5,463  | 5,363  | 5,304  | 5,500  |

<===== (diagonal) x (age -> ult)

## Step 3a calculate ultimate losses as:

(ultimate counts) x (ultimate severities)

|                        |       |       |       |       |        |              |
|------------------------|-------|-------|-------|-------|--------|--------------|
| ultimate counts        | 399   | 350   | 295   | 260   | <===== | from Step 1  |
| ultimate severities    | 5,463 | 5,363 | 5,304 | 5,500 | <===== | from Step 2b |
| ultimate losses (000s) | 2,182 | 1,875 | 1,565 | 1,430 |        |              |

## Step 3b calculate unpaid losses for AY 2025

<==== pay attention to whether the question asks for **ultimate** or **unpaid**

|        |   |          |   |                  |
|--------|---|----------|---|------------------|
| unpaid | = | ultimate | - | latest paid      |
|        | = | 2,182    | - | 700              |
|        | = | 1,482    |   | <-- final answer |

**Reading:** Friedland 12 (Case Outstanding Methods)  
**Model:** Basic Practice  
**Problem Type:** Reserving Methods - COS (Case O/S Method #2 - Formula)

## Reserving-B: Case OS Method (Problem)

**Find** Calculate the unpaid claim estimate for this self-insured entity for AY

2024

**Given** Information as of Dec 31, 2025:

|                      |       |                     |
|----------------------|-------|---------------------|
| case O/S for AY 2024 | 1,800 | <===== company data |
|----------------------|-------|---------------------|

| industry data |     |  | paid | reported |
|---------------|-----|--|------|----------|
| 12 - ultimate | CDF |  | 6.25 | 3.13     |
| 24 - ultimate | CDF |  | 1.86 | 1.37     |
| 36 - ultimate | CDF |  | 1.35 | 1.21     |
| 48 - ultimate | CDF |  | 1.24 | 1.08     |
| 60 - ultimate | CDF |  | 1.16 | 1.05     |
| 72 - ultimate | CDF |  | 1.09 | 1.03     |

Step 1 highlight the correct CDFs for the given AY

| AY   | industry CDFs | paid | reported |
|------|---------------|------|----------|
| 2020 | 72 - ultimate | 1.09 | 1.03     |
| 2021 | 60 - ultimate | 1.16 | 1.05     |
| 2022 | 48 - ultimate | 1.24 | 1.08     |
| 2023 | 36 - ultimate | 1.35 | 1.21     |
| 2024 | 24 - ultimate | 1.86 | 1.37     |
| 2025 | 12 - ultimate | 6.25 | 3.13     |

<===== use these CDFs

Step 2 substitute the correct CDFs into the case O/S formula for estimating unpaid for AY

2024

$$\begin{aligned}\text{case O/S factor} &= 1 + \frac{[(\text{rptd:CDF} - 1) \times \text{paid:CDF}]}{(\text{paid:CDF} - \text{rptd:CDF})} \\ &= 1 + \frac{[(1.37 - 1) \times 1.86]}{(1.86 - 1.37)} \\ &= 1 + \frac{0.6882}{0.4900} \\ &= 2.4045\end{aligned}$$

$$\begin{aligned}\text{unpaid} &= \text{case O/S amount} \times \text{case O/S factor} \\ &= 1,800 \times 2.4045 \\ &= 4,328 \\ &\text{(final answer)}\end{aligned}$$

**Reading:** Friedland 13 (Berquist-Sherman Methods)  
**Model:** 2019.Fall #21  
**Problem Type:** Reserving Methods - RBS (Reported Berq-Sherm)

## Reserving-B: RBS Method (Problem)

**Find** Use the reported Berquist-Sherman method to calculate the

unpaid for AY 2025

**Given**

| AY   | Cumulative Paid <u>Loss</u> (\$000s) <b>(CPL)</b> |       |       |       |
|------|---------------------------------------------------|-------|-------|-------|
|      | 12                                                | 24    | 36    | 48    |
| 2022 | 560                                               | 1,325 | 1,650 | 1,680 |
| 2023 | 650                                               | 1,350 | 1,720 |       |
| 2024 | 615                                               | 1,305 |       |       |
| 2025 | 625                                               |       |       |       |

| AY   | Cumulative Reported <u>Loss</u> (\$000s) <b>(CRL)</b> |       |       |       |
|------|-------------------------------------------------------|-------|-------|-------|
|      | 12                                                    | 24    | 36    | 48    |
| 2022 | 1,100                                                 | 1,650 | 1,675 | 1,680 |
| 2023 | 1,250                                                 | 1,680 | 1,750 |       |
| 2024 | 1,200                                                 | 1,800 |       |       |
| 2025 | 1,500                                                 |       |       |       |

| AY   | Cumulative Paid <u>Counts</u> <b>(CPC)</b> |     |     |     |
|------|--------------------------------------------|-----|-----|-----|
|      | 12                                         | 24  | 36  | 48  |
| 2022 | 78                                         | 106 | 114 | 115 |
| 2023 | 80                                         | 111 | 118 |     |
| 2024 | 75                                         | 99  |     |     |
| 2025 | 82                                         |     |     |     |

| AY   | Cumulative Reported <u>Counts</u> <b>(CRC)</b> |     |     |     |
|------|------------------------------------------------|-----|-----|-----|
|      | 12                                             | 24  | 36  | 48  |
| 2022 | 108                                            | 115 | 115 | 115 |
| 2023 | 112                                            | 120 | 120 |     |
| 2024 | 104                                            | 110 |     |     |
| 2025 | 106                                            |     |     |     |

|                       |      |
|-----------------------|------|
| annual severity trend | 5.0% |
|-----------------------|------|

Step 1a calculate Average Paid Loss (APL) & Average Case O/S (ACOS)

Reserving-B: RBS Method (Solution)

| AY   | Average Paid Loss (\$000s) (APL) |      |      |      |
|------|----------------------------------|------|------|------|
|      | 12                               | 24   | 36   | 48   |
| 2022 | 7.2                              | 12.5 | 14.5 | 14.6 |
| 2023 | 8.1                              | 12.2 | 14.6 |      |
| 2024 | 8.2                              | 13.2 |      |      |
| 2025 | 7.6                              |      |      |      |

Ex:  $8.1 = 650 / 80$   
 $APL = CPL / CPC$

| AY   | Average Case O/S (\$000s) (ACOS) |      |      |     |
|------|----------------------------------|------|------|-----|
|      | 12                               | 24   | 36   | 48  |
| 2022 | 18.0                             | 36.1 | 25.0 | 0.0 |
| 2023 | 18.8                             | 36.7 | 15.0 |     |
| 2024 | 20.2                             | 45.0 |      |     |
| 2025 | 36.5                             |      |      |     |

Ex:  $18.8 = (1250 - 650) / (112 - 80)$   
 $ACOS = (CRL - CPL) / (CRC - CPC)$

Step 1b calculate & select severity trend down columns (if trend is not provided)

| AY   | Average Paid Loss TREND |       |      |    |
|------|-------------------------|-------|------|----|
|      | 12                      | 24    | 36   | 48 |
| 2022 |                         |       |      |    |
| 2023 | 12.5%                   | -2.4% | 0.7% |    |
| 2024 | 1.2%                    | 8.2%  |      |    |
| 2025 | -7.3%                   |       |      |    |

severity trend selection: n/a  
 severity trend given: 5.0%

| AY   | Average Case O/S TREND |       |        |    |
|------|------------------------|-------|--------|----|
|      | 12                     | 24    | 36     | 48 |
| 2022 |                        |       |        |    |
| 2023 | 4.4%                   | 1.7%  | -40.0% |    |
| 2024 | 7.4%                   | 22.6% |        |    |
| 2025 | 80.7%                  |       |        |    |

Note increase along latest diagonal.  
 (Evidence for case reserve strengthening in CY 2025)

Step 2a restate ACOS by de-trending from latest diagonal of original ACOS

| AY   | Average Case O/S (ACOS) |      |      |     |
|------|-------------------------|------|------|-----|
|      | 12                      | 24   | 36   | 48  |
| 2022 | 31.5                    | 40.8 | 14.3 | 0.0 |
| 2023 | 33.1                    | 42.9 | 15.0 |     |
| 2024 | 34.8                    | 45.0 |      |     |
| 2025 | 36.5                    |      |      |     |

\* latest diagonal doesn't change under de-trending

Ex:  $33.1 = 36.5 / 1.05^2$

Step 2b restate CRL using restated ACOS from step 2a: (restated ACOS) x (open counts) + (original CPL)

| AY   | Restated CRL |       |       |       |
|------|--------------|-------|-------|-------|
|      | 12           | 24    | 36    | 48    |
| 2022 | 1,505        | 1,692 | 1,664 | 1,680 |
| 2023 | 1,709        | 1,736 | 1,750 |       |
| 2024 | 1,624        | 1,800 |       |       |
| 2025 | 1,500        |       |       |       |

Ex:  $1,709 = 33.1 \times (112 - 80) + 650$

Step 3b apply development method to restated CRL from step 2b

| LDFs     | AY   | 12-24 | 24-36 | 36-48 | 48-ult |
|----------|------|-------|-------|-------|--------|
|          | 2022 | 1.124 | 0.984 | 1.009 |        |
|          | 2023 | 1.016 | 1.008 |       |        |
|          | 2024 | 1.108 |       |       |        |
|          | 2025 |       |       |       |        |
| selected |      | 1.083 | 0.996 | 1.009 | 1.000  |

Tail Factor: Set = 1.0 if triangle is fully developed.  
 If triangle is not fully developed then select something different from 1.0

| CDFs       |  | 12-ult | 24-ult | 36-ult | 48-ult |
|------------|--|--------|--------|--------|--------|
| age -> ult |  | 1.088  | 1.005  | 1.009  | 1.000  |

<===== (selected) x (prior [age -> ult])  
 (calculate from right-to-left)

| Ults.    |  | '25@12 | '24@24 | '23@36 | '22@48 |
|----------|--|--------|--------|--------|--------|
| diagonal |  | 1,500  | 1,800  | 1,750  | 1,680  |
| ultimate |  | 1,632  | 1,809  | 1,766  | 1,680  |

<===== (diagonal) x (age -> ult)

Step 3b unpaid for AY 2025 = 1,007 <-- final answer

**Reading:** Friedland 05 (Triangles)  
**Model:** 2014.Fall #14  
**Problem Type:** Building Triangles from Raw Claims Data

## Reserving-C: Building RY Triangles (Problem)

**Find** Build the **RY (Report Year)** paid & reported claim triangles. (Unless stated otherwise these are cumulative triangles)

**Given**

| claim id | AY   | RY   | <u>2020</u> |          | <u>2021</u> |          | <u>2022</u> |          |
|----------|------|------|-------------|----------|-------------|----------|-------------|----------|
|          |      |      | paid in     | ending   | paid in     | ending   | paid in     | ending   |
|          |      |      | CY          | case O/S | CY          | case O/S | CY          | case O/S |
| 1        | 2020 | 2020 | 800         | 800      | 700         | 300      | 600         | 100      |
| 2        | 2020 | 2020 | 700         | 0        | 0           | 0        | 0           | 0        |
| 3        | 2020 | 2020 | 700         | 500      | 500         | 400      | 500         | 200      |
| 4        | 2020 | 2020 | 300         | 600      | 800         | 200      | 300         | 100      |
| 5        | 2020 | 2020 | 900         | 500      | 500         | 400      | 500         | 200      |
| 6        | 2021 | 2021 | -           | -        | 900         | 300      | 400         | 200      |
| 7        | 2021 | 2021 | -           | -        | 800         | 200      | 400         | 200      |
| 8        | 2021 | 2022 | -           | -        | -           | -        | 500         | 100      |
| 9        | 2022 | 2022 | -           | -        | -           | -        | 600         | 0        |
| 10       | 2022 | 2023 | -           | -        | -           | -        | -           | -        |
| 11       | 2022 | 2023 | -           | -        | -           | -        | -           | -        |
| 12       | 2022 | 2023 | -           | -        | -           | -        | -           | -        |
| 13       | -    | -    | -           | -        | -           | -        | -           | -        |
| 14       | -    | -    | -           | -        | -           | -        | -           | -        |
| 15       | -    | -    | -           | -        | -           | -        | -           | -        |
| 16       | -    | -    | -           | -        | -           | -        | -           | -        |
| 17       | -    | -    | -           | -        | -           | -        | -           | -        |
| 18       | -    | -    | -           | -        | -           | -        | -           | -        |

## Reserving-C: Building RY Triangles (Solution)

**Step 1** Sum across **RY** (This is simple in Excel but easy to make a mistake with pencil and paper.)

**Suggestion:** Use a ruler to draw a horizontal line between one AY and the next.

| AY  | RY   | <u>2020</u>   |                          | <u>2021</u>   |                          | <u>2022</u>   |                          |
|-----|------|---------------|--------------------------|---------------|--------------------------|---------------|--------------------------|
|     |      | paid in<br>CY | ending<br>case O/S<br>CY | paid in<br>CY | ending<br>case O/S<br>CY | paid in<br>CY | ending<br>case O/S<br>CY |
| all | 2020 | 3,400         | 2,400                    | 2,500         | 1,300                    | 1,900         | 600                      |
| all | 2021 | 0             | 0                        | 1,700         | 500                      | 800           | 400                      |
| all | 2022 | 0             | 0                        | 0             | 0                        | 1,100         | 100                      |
| all | 2023 | 0             | 0                        | 0             | 0                        | 0             | 0                        |

**Step 2** Transfer the colour-coded values from Step 1 into these triangles...

| incremental PAID claims |       |       |       | case O/S |       |       |     |
|-------------------------|-------|-------|-------|----------|-------|-------|-----|
| RY                      | 12    | 24    | 36    | RY       | 12    | 24    | 36  |
| 2020                    | 3,400 | 2,500 | 1,900 | 2020     | 2,400 | 1,300 | 600 |
| 2021                    | 1,700 | 800   |       | 2021     | 500   | 400   |     |
| 2022                    | 1,100 |       |       | 2022     | 100   |       |     |
| 2023                    |       |       |       | 2023     |       |       |     |

**Step 3** Form the CUMULATIVE paid triangle by summing across rows in the incremental paid triangle

(final  
answer)

| cumulative PAID claims |       |       |       |
|------------------------|-------|-------|-------|
| RY                     | 12    | 24    | 36    |
| 2020                   | 3,400 | 5,900 | 7,800 |
| 2021                   | 1,700 | 2,500 |       |
| 2022                   | 1,100 |       |       |
| 2023                   |       |       |       |

Ex:  $5,900 = 3,400 + 2,500$   
 $7,800 = 3,400 + 2,500 + 1,900$

**Step 4** Form the CUMULATIVE reported triangle:  $\text{cumulative reported claims} = (\text{cumulative paid claims}) + (\text{Case O/S})$

(final  
answer)

| cumulative REPORTED claims |       |       |       |
|----------------------------|-------|-------|-------|
| RY                         | 12    | 24    | 36    |
| 2020                       | 5,800 | 7,200 | 8,400 |
| 2021                       | 2,200 | 2,900 |       |
| 2022                       | 1,200 |       |       |
| 2023                       |       |       |       |



**Reading:** Friedland 07 (Development Method)  
**Model:** 2017.Fall #19  
**Problem Type:** Reported Claims Development

## Reserving-C: Tail Factors (Problem)

- Find**
- a** Calculate ultimate claims for AY 2022 and 2023 using data as of year-end 2023
  - b** Calculate a diagnostic showing a recent operational change and describe a likely scenario

**Given**

**cumulative reported claims**

| AY   | 12    | 24    | 36    | 48    |
|------|-------|-------|-------|-------|
| 2020 | 1,440 | 2,625 | 3,240 | 3,476 |
| 2021 | 1,385 | 2,589 | 3,003 |       |
| 2022 | 1,333 | 2,556 |       |       |
| 2023 | 1,286 |       |       |       |

**cumulative paid claims**

| AY   | 12  | 24    | 36    | 48    |
|------|-----|-------|-------|-------|
| 2020 | 900 | 2,232 | 3,070 | 3,414 |
| 2021 | 878 | 2,213 | 2,755 |       |
| 2022 | 857 | 2,195 |       |       |
| 2023 | 837 |       |       |       |

**reported claims development technique ultimates:**

| AY   | amt   |
|------|-------|
| 2020 | 3,600 |
| 2021 | 3,337 |

## Reserving-C: Tail Factors Method (Solution)

### a link ratios for reported claim triangle

| AY       | 12-24 | 24-36 | 36-48 | 48-ult |
|----------|-------|-------|-------|--------|
| 2020     | 1.823 | 1.234 | 1.073 |        |
| 2021     | 1.870 | 1.160 |       |        |
| 2022     | 1.917 |       |       |        |
| 2023     |       |       |       |        |
| selected | 1.870 | 1.197 | 1.073 | 1.036  |

**Trick:** The triangle is not fully developed as of 48 months. You must calculate a tail factor using AY 2020 information as shown below.

$$= \frac{3,600}{3,476}$$

AY 2020 ultimate      AY 12 @48 months

### calculate age-to-ultimate LDFs

|            | 12-ult | 24-ult | 36-ult | 48-ult |
|------------|--------|--------|--------|--------|
| age -> ult | 2.487  | 1.330  | 1.111  | 1.036  |

$$\text{<===== (selected) x (prior [age -> ult])}$$

(calculate from right-to-left)

### calculate ultimate losses based on latest reported losses

|          | '23@12 | '22@24 | '21@36 | '20@48 |
|----------|--------|--------|--------|--------|
| diagonal | 1,286  | 2,556  | 3,003  | 3,476  |
| ultimate | 3,198  | 3,400  | 3,337  | 3,600  |

$$\text{<===== (diagonal) x (age -> ult)}$$

### Final answers:

AY 2022 ultimate loss based on reported losses: 3,400  
 AY 2023 ultimate loss based on reported losses: 3,198

### b Since we're only given cumulative reported and paid triangles, we don't have many options for diagnostics. We could calculate incremental triangles, but we'll first try the obvious and calculate paid / reported.

### cumulative paid / reported

| AY   | 12   | 24   | 36   | 48   |
|------|------|------|------|------|
| 2020 | 0.63 | 0.85 | 0.95 | 0.98 |
| 2021 | 0.63 | 0.85 | 0.92 |      |
| 2022 | 0.64 | 0.86 |      |      |
| 2023 | 0.65 |      |      |      |

Looking down the columns, we see there has been a general increase in the paid/reported ratio

This likely means: (paid values went up) OR (reported values went down) OR (a combination of both)

Possible scenarios corresponding to these observations are:

- [1] claims handling rules were LOOSENED
- [2] case reserve adequacy DECREASED

**Reading:** Friedland 09 (Bornhuetter-Ferguson Method)  
**Model:** 2019.Spring #19  
**Problem Type:** Reserving Methods - BF with Tort Reform

## Reserving-C: BF Method - Tort Reform (Problem)

**Find** Calculate the ultimate for AY 2024 using:

- (i) reported development method
- (ii) reported Bornhuetter-Ferguson method

**Given** Data is for CY/AY 2024 as of Dec 31, 2024

|                           |       |
|---------------------------|-------|
| EP                        | 2,000 |
| ECR                       | 87%   |
| reported loss @ 12 months | 1,120 |
| reported CDF for 12-ult   | 2.300 |
| paid loss @ 12 months     | 460   |

Tort reform is passed on Dec 31, 2024 causing an expected 20% reduction on future claim payments.

## (i) reported development method

Step 1 reported development estimate without tort reform adjustment

$$\begin{aligned}
 &= \text{rptd loss} \quad \times \quad \text{CDF} \\
 &= 1,120 \quad \times \quad 2.300 \\
 &= 2,576
 \end{aligned}$$

Step 2 unpaid claims (*need this because we make the tort reform adjustment only to unpaid claims*)

$$\begin{aligned}
 &= \text{ultimate} \quad - \quad \text{paid loss} \\
 &= 2,576 \quad - \quad 460 \\
 &= 2,116
 \end{aligned}$$

Step 3 adjust unpaid loss downward by:

20%

adjusted unpaid loss

$$\begin{aligned}
 &= 2,116 \quad \times \quad 80\% \\
 &= 1,693
 \end{aligned}$$

Step 4 adjusted ultimate

$$\begin{aligned}
 &= \text{adjusted unpaid loss} \quad + \quad \text{paid loss} \\
 &= 1,693 \quad + \quad 460 \\
 &= 2,153 \quad \text{<== final answer}
 \end{aligned}$$

## (ii) reported Bornhuetter-Ferguson method

\* Steps 2,3,4 are the same as for part (i)

Step 1a ECR method estimate without tort reform adjustment

$$\begin{aligned}
 &= \text{ECR} \quad \times \quad \text{EP} \\
 &= 87\% \quad + \quad 2,000 \\
 &= 1,740
 \end{aligned}$$

Step 1b BF method estimate without tort reform adjustment

$$\begin{aligned}
 &= \text{rptd loss} \quad + \quad (1 - 1/\text{CDF}) \quad \times \quad \text{ECR ult} \\
 &= 1,120 \quad + \quad 0.565 \quad \times \quad 1,740 \\
 &= 2,103
 \end{aligned}$$

Step 2 unpaid claims (*need this because we make the tort reform adjustment only to unpaid claims*)

$$\begin{aligned}
 &= \text{ultimate} \quad - \quad \text{paid loss} \\
 &= 2,103 \quad - \quad 460 \\
 &= 1,643
 \end{aligned}$$

Step 3 adjust unpaid loss downward by:

20%

adjusted unpaid loss

$$\begin{aligned}
 &= 1,643 \quad \times \quad 80\% \\
 &= 1,315
 \end{aligned}$$

Step 4 adjusted ultimate

$$\begin{aligned}
 &= \text{adjusted unpaid loss} \quad + \quad \text{paid loss} \\
 &= 1,315 \quad + \quad 460 \\
 &= 1,775 \quad \text{<== final answer}
 \end{aligned}$$

**Reading:** Friedland 10 (Cape Cod Method)

**Model:** 2018.Spring #18

**Problem Type:** Reserving Methods - CC (prior AYs) with Pure Premium Trend Adjustments

Reserving-C: CC Method - Trends (Problem)

Find

Calculate the

ultimate

for AY

2024

using the reported Cape Cod method.

Given

| AY   | rptd clms | CDF  | pure premium trend | CY   | EP    |
|------|-----------|------|--------------------|------|-------|
| 2023 | 2,900     | 2.30 | 1.067              | 2023 | 6,500 |
| 2024 | 1,800     | 3.90 | 0.983              | 2024 | 8,100 |
| 2025 | 1,000     | 7.60 | 1.000              | 2025 | 8,000 |

\* no rate level adjustments

\* no premium trend

## Reserving-C: CC Method - Trends (Solution)

**Step 1** calculate the adjusted reported losses (*do not apply CDFs to the losses - we are not calculating the ultimate here*)

| AY   | rptd clms | x | PP trend | = | adjusted reported claims |
|------|-----------|---|----------|---|--------------------------|
| 2023 | 2,900     | x | 1.067    | = | 3,094                    |
| 2024 | 1,800     | x | 0.983    | = | 1,769                    |
| 2025 | 1,000     | x | 1.000    | = | 1,000                    |
|      |           |   |          |   | <b>5,864</b>             |

**Step 2** calculate the used-up-premium (UUP)

| CY/AY | EP    | / | CDF  | = | UUP          |
|-------|-------|---|------|---|--------------|
| 2023  | 6,500 | / | 2.30 | = | 2,826        |
| 2024  | 8,100 | / | 3.90 | = | 2,077        |
| 2025  | 8,000 | / | 7.60 | = | 1,053        |
|       |       |   |      |   | <b>5,956</b> |

**Step 3** calculate the adjusted ECR at the level: 2024 (*you must back out the pure premium trend*)

$$\begin{aligned}
 \text{ECR} &= \text{Step 1} / \text{Step 2} / (\text{PP trend for AY 2024}) \\
 &= 5,864 / 5,956 / 0.983 \\
 &= 100.16\%
 \end{aligned}$$

**Step 4** calculate the ultimate using the data for: 2024

$$\begin{aligned}
 \text{ultimate} &= \text{rptd clms} + (1 - 1 / \text{CDF}) \times \text{ECR} \times \text{EP} \\
 &= 1,800 + (1 - 1 / 3.9) \times 100.16\% \times 8,100 \\
 &= 7,833 \quad \text{<== final answer}
 \end{aligned}$$

**Reading:** Friedland 10 (Frequency-Severity Methods)  
**Model:** 2019.Fall #20  
**Problem Type:** Reserving Methods - FS (Disposal Rate Method)

## Reserving-D: FS Method - Disposal Rates (Problem)

**Find** Use the frequency-severity disposal rate method to estimate **unpaid** claims for AY 2025  
**Data Set:**

**Given** A court ruling on Dec 31, 2025 will increase future claims payments by **20%**  
 All claims are closed by 48 months  
 Severity trend: **0%**

| AY   | <u>Cumulative</u> Paid Counts <b>(CPC)</b> |     |     |     | Ultimate<br>Counts |
|------|--------------------------------------------|-----|-----|-----|--------------------|
|      | 12                                         | 24  | 36  | 48  |                    |
| 2022 | 308                                        | 555 | 642 | 647 | 647                |
| 2023 | 356                                        | 563 | 678 |     | 683                |
| 2024 | 358                                        | 575 |     |     | 684                |
| 2025 | 402                                        |     |     |     | 795                |

| AY   | <u>Cumulative</u> Paid Loss (\$000s) <b>(CPL)</b> |     |     |     |
|------|---------------------------------------------------|-----|-----|-----|
|      | 12                                                | 24  | 36  | 48  |
| 2022 | 375                                               | 745 | 906 | 916 |
| 2023 | 397                                               | 750 | 922 |     |
| 2024 | 422                                               | 762 |     |     |
| 2025 | 385                                               |     |     |     |

**Step 1a** calculate CPC and IPC

(you will be given 1 of them and have to calculate the other)

| given | Cumulative Paid Counts (CPC) |     |     |     |
|-------|------------------------------|-----|-----|-----|
| AY    | 12                           | 24  | 36  | 48  |
| 2022  | 308                          | 555 | 642 | 647 |
| 2023  | 356                          | 563 | 678 |     |
| 2024  | 358                          | 575 |     |     |
| 2025  | 402                          |     |     |     |

**Reserving-D: FS Method - Disposal Rates (Solution)**

|      | Incremental Paid Counts (IPC) |     |     |    |
|------|-------------------------------|-----|-----|----|
| AY   | 12                            | 24  | 36  | 48 |
| 2022 | 308                           | 247 | 87  | 5  |
| 2023 | 356                           | 207 | 115 |    |
| 2024 | 358                           | 217 |     |    |
| 2025 | 402                           |     |     |    |

**Step 1b** calculate CPL and IPL

(you will be given 1 of them and have to calculate the other)

| given | Cumulative Paid Loss (\$000s) (CPL) |     |     |     |
|-------|-------------------------------------|-----|-----|-----|
| AY    | 12                                  | 24  | 36  | 48  |
| 2022  | 375                                 | 745 | 906 | 916 |
| 2023  | 397                                 | 750 | 922 |     |
| 2024  | 422                                 | 762 |     |     |
| 2025  | 385                                 |     |     |     |

|      | Incremental Paid Loss (\$000s) (IPL) |     |     |    |
|------|--------------------------------------|-----|-----|----|
| AY   | 12                                   | 24  | 36  | 48 |
| 2022 | 375                                  | 370 | 161 | 10 |
| 2023 | 397                                  | 353 | 172 |    |
| 2024 | 422                                  | 340 |     |    |
| 2025 | 385                                  |     |     |    |

**Step 2a** calculate CDR as:  $CPC / UC$  (UC = Ultimate Counts)

|               | Claims Disposal Rate (CDR) |              |              |              |
|---------------|----------------------------|--------------|--------------|--------------|
| AY            | 12                         | 24           | 36           | 48           |
| 2022          | 0.476                      | 0.858        | 0.992        | 1.000        |
| 2023          | 0.521                      | 0.824        | 0.993        |              |
| 2024          | 0.523                      | 0.841        |              |              |
| 2025          | 0.506                      |              |              |              |
| <b>select</b> | <b>0.507</b>               | <b>0.841</b> | <b>0.993</b> | <b>1.000</b> |

<-- default selection = all period average

**Step 2b** project IPC

(project the Incremental Paid Counts to the lower right portion of the IPC triangle)

|      | Incremental Paid Counts (IPC) |     |     |    |
|------|-------------------------------|-----|-----|----|
| AY   | 12                            | 24  | 36  | 48 |
| 2022 |                               |     |     | 5  |
| 2023 |                               |     |     |    |
| 2024 |                               |     | 104 | 5  |
| 2025 |                               | 266 | 121 | 6  |

This is the part of the calculation where it's easy to make a mistake.

You have to distribute the remaining counts proportionately using the disposal rate you selected in the previous step.

|            |     |   | remaining counts |   |                   | proportional distribution |               |
|------------|-----|---|------------------|---|-------------------|---------------------------|---------------|
| 2025/24:   | 266 | = | ( 795 - 402 )    | x | ( 0.841 - 0.507 ) | /                         | ( 1 - 0.507 ) |
| 2025/36:   | 121 | = | ( 795 - 402 )    | x | ( 0.993 - 0.841 ) | /                         | ( 1 - 0.507 ) |
| 2025/48:   | 6   | = | ( 795 - 402 )    | x | ( 1 - 0.993 )     | /                         | ( 1 - 0.507 ) |
| * 2024/36: | 104 | = | ( 684 - 575 )    | x | ( 0.993 - 0.841 ) | /                         | ( 1 - 0.841 ) |
| * 2024/48: | 5   | = | ( 684 - 575 )    | x | ( 1 - 0.993 )     | /                         | ( 1 - 0.841 ) |
| * 2023/48: | 5   | = | ( 683 - 678 )    | x | ( 1 - 0.993 )     | /                         | ( 1 - 0.993 ) |

\* Note that don't have to project every AY. You only have to project the rows for AYs you're asked to calculate.

**Step 3a** calculate IPS trended to AY 2025, and select an AY 2025 severity for the unpaid periods 24, 36, 48

|             | Incremental Paid Severity (IPS) |              |              |              |
|-------------|---------------------------------|--------------|--------------|--------------|
| AY          | 12                              | 24           | 36           | 48           |
| 2022        | 1.218                           | 1.498        | 1.851        | 2.000        |
| 2023        | 1.115                           | 1.705        | 1.496        |              |
| 2024        | 1.179                           | 1.567        |              |              |
| <b>2025</b> | <b>0.958</b>                    | <b>1.590</b> | <b>1.673</b> | <b>2.000</b> |

demonstration of severity calc & trending for age 24

|       |   |                              |
|-------|---|------------------------------|
| 1.498 | = | ( 370 / 247 ) x ( 1 + 0% )^3 |
| 1.705 | = | ( 353 / 207 ) x ( 1 + 0% )^2 |
| 1.567 | = | ( 340 / 217 ) x ( 1 + 0% )^1 |

<-- default selected severity = all period average

counts 266 121 6 <-- from Step 2b

**Step 3b** unpaid <-- (selected severity) x counts

court ruling adj. 20% 20% 20% <-- adjust unpaid by this percentage

adj. unpaid 508 243 14 <-- final answer for UNPAID loss

1,150 <-- ULTIMATE loss



**Reading:** Friedland 12 (Case Outstanding Methods)  
**Model:** 2019.Fall #18  
**Problem Type:** Reserving Methods - COS (Case O/S Method #1)

## Reserving-D: Case OS Method 1 (Problem)

**Find:** Use a Case O/S method to calculate the unpaid for AY 2025  
**Data Set:** Book of Triangles *Scenario 0: stable data*

**Given:**

| AY   | Cumulative Paid Loss (\$000s) (CPL) |       |       |       |
|------|-------------------------------------|-------|-------|-------|
|      | 12                                  | 24    | 36    | 48    |
| 2022 | 720                                 | 1,800 | 2,880 | 3,600 |
| 2023 | 720                                 | 1,800 | 2,880 |       |
| 2024 | 720                                 | 1,800 |       |       |
| 2025 | 720                                 |       |       |       |

| AY   | Case Outstanding (\$000s) (COS) |       |     |     |
|------|---------------------------------|-------|-----|-----|
|      | 12                              | 24    | 36  | 48  |
| 2022 | 1,440                           | 1,080 | 720 | 360 |
| 2023 | 1,440                           | 1,080 | 720 |     |
| 2024 | 1,440                           | 1,080 |     |     |
| 2025 | 1,440                           |       |     |     |

|                                                |      |
|------------------------------------------------|------|
| 48-ultimate paid loss to prior case O/S factor | 1.00 |
|------------------------------------------------|------|

\* no development past 60 months

Step 1 calculate IPL (Incremental Paid Loss)

| AY   | Incremental Paid Loss (\$000s) (IPL) |       |       |     |
|------|--------------------------------------|-------|-------|-----|
|      | 12                                   | 24    | 36    | 48  |
| 2022 | 720                                  | 1,080 | 1,080 | 720 |
| 2023 | 720                                  | 1,080 | 1,080 |     |
| 2024 | 720                                  | 1,080 |       |     |
| 2025 | 720                                  |       |       |     |

Reserving-D: Case OS Method 1 (Solution)

Sometimes you're given the incremental paid loss.  
If so, then step 1 is not necessary.

Step 2a calculate & make selections for: (current case) / (prior case)

| AY   | Current/Prior COS |       |       |       |
|------|-------------------|-------|-------|-------|
|      | 24/12             | 36/24 | 48/36 | 60/48 |
| 2022 | 0.750             | 0.667 | 0.500 |       |
| 2023 | 0.750             | 0.667 |       |       |
| 2024 | 0.750             |       |       |       |
| 2025 | 0.750             | 0.667 | 0.500 | 0.000 |

Demonstration for AY 2022 calculations:

|       | current case O/S |   | prior case O/S |
|-------|------------------|---|----------------|
| 24/12 | 0.750            | = | 1,080 / 1,440  |
| 36/24 | 0.667            | = | 720 / 1,080    |
| 48/36 | 0.500            | = | 360 / 720      |

<-- default selection = all period average

Step 2b project case O/S using selections from step 2a

| AY   | projected Case Outstanding (COS) |       |     |     |
|------|----------------------------------|-------|-----|-----|
|      | 12                               | 24    | 36  | 48  |
| 2022 |                                  |       |     | 360 |
| 2023 |                                  |       | 720 | 360 |
| 2024 |                                  | 1,080 | 720 | 360 |
| 2025 | 1,440                            | 1,080 | 720 | 360 |

Demonstration for AY 2025 projections:

|         | projection | = | selection |   | prior case O/S |
|---------|------------|---|-----------|---|----------------|
| 24 mths | 1,080      | = | 0.750     | x | 1,440          |
| 36 mths | 720        | = | 0.667     | x | 1,080          |
| 48 mths | 360        | = | 0.500     | x | 720            |

Step 3a calculate & make selections for: (current IPL) / (prior case)

| AY   | IPL / (prior case) |       |       |       |
|------|--------------------|-------|-------|-------|
|      | 24/12              | 36/24 | 48/36 | 60/48 |
| 2022 | 0.750              | 1.000 | 1.000 |       |
| 2023 | 0.750              | 1.000 |       |       |
| 2024 | 0.750              |       |       |       |
| 2025 | 0.750              | 1.000 | 1.000 |       |

Demonstration for AY 2022 calculations:

|       | current IPL |   | prior case O/S |
|-------|-------------|---|----------------|
| 24/12 | 0.750       | = | 1,080 / 1,440  |
| 36/24 | 1.000       | = | 1,080 / 1,080  |
| 48/36 | 1.000       | = | 720 / 720      |

<-- default selection = all period average

Step 3b project IPL using selections from step 3a

| AY   | projected Incremental Paid Loss (IPL) |       |       |     |     |
|------|---------------------------------------|-------|-------|-----|-----|
|      | 12                                    | 24    | 36    | 48  | 60  |
| 2022 |                                       |       |       |     | 360 |
| 2023 |                                       |       |       | 720 | 360 |
| 2024 |                                       |       | 1,080 | 720 | 360 |
| 2025 |                                       | 1,080 | 1,080 | 720 | 360 |

Demonstration for AY 2025 projections:

|         | projection | = | selection * |   | prior case O/S |
|---------|------------|---|-------------|---|----------------|
| 24 mths | 1,080      | = | 0.750       | x | 1,440          |
| 36 mths | 1,080      | = | 1.000       | x | 1,080          |
| 48 mths | 720        | = | 1.000       | x | 720            |
| 60 mths | 360        | = | 1.000       | x | 360            |

**Note:** The IPL projection always has 1 extra column. In this case 60-months. We use the 48-month case O/S to project the IPL out to 60 months.

\* The 60-mth selection was given info.  
(It was not selected in step 3a)

Final calculate ultimate & unpaid amounts:

unpaid = sum of rows in step 3b  
ultimate = sum of rows in step 3b + last diagonal of CPL

| AY   | ultimate | unpaid |
|------|----------|--------|
| 2022 | 3,960    | 360    |
| 2023 | 3,960    | 1,080  |
| 2024 | 3,960    | 2,160  |
| 2025 | 3,960    | 3,240  |

Pay attention to whether the question asks for ultimate or unpaid.  
Also, pay attention to which AY is being asked about.

The final unpaid for AY 2025 is 3,240 <-- final answer

**Reading:** Friedland 13 (Berquist-Sherman Methods)  
**Model:** 2016.Spring #20  
**Problem Type:** Reserving Methods - PBS (Paid Berq-Sherm)

## Reserving-D: PBS Method (Problem)

**Find** Use the paid Berquist-Sherman method to calculate the ultimate for AY 2025 using both linear interpolation and exponential regression to restate the cumulative paid losses.

**Given**

| AY   | Cumulative Paid Counts (CPC) |       |       |       | Ultimate Counts |
|------|------------------------------|-------|-------|-------|-----------------|
|      | 12                           | 24    | 36    | 48    |                 |
| 2022 | 3,314                        | 4,260 | 4,340 | 4,380 | 4,380           |
| 2023 | 3,390                        | 4,404 | 4,550 |       | 4,596           |
| 2024 | 3,342                        | 4,365 |       |       | 4,454           |
| 2025 | 3,607                        |       |       |       | 4,509           |

*\* Assume no development past 48 mths.*

| AY   | Cumulative Paid Loss (\$000s) (CPL) |        |        |        |
|------|-------------------------------------|--------|--------|--------|
|      | 12                                  | 24     | 36     | 48     |
| 2022 | 7,760                               | 13,664 | 15,515 | 16,484 |
| 2023 | 8,797                               | 13,543 | 16,824 |        |
| 2024 | 7,821                               | 13,928 |        |        |
| 2025 | 9,113                               |        |        |        |

*\* Assume no development past 48 mths.*

| AY   | parameters for 2-point exponential fit: $y = ae^{bx}$ |   |         |         |         |         |         |         |            |   |
|------|-------------------------------------------------------|---|---------|---------|---------|---------|---------|---------|------------|---|
|      | 0 - 12                                                |   | 12 - 24 |         | 24 - 36 |         | 36 - 48 |         | 48 - 60    |   |
|      | a                                                     | b | a       | b       | a       | b       | a       | b       | a          | b |
| 2022 | use 12 - 24 values                                    |   | 1,069   | 0.00060 | 16      | 0.00159 | 22      | 0.00152 | not needed |   |
| 2023 | use 12 - 24 values                                    |   | 2,079   | 0.00043 | 19      | 0.00149 |         |         |            |   |
| 2024 | use 12 - 24 values                                    |   | 1,187   | 0.00056 |         |         |         |         |            |   |
| 2025 | use 12 - 24 values                                    |   |         |         |         |         |         |         |            |   |

*\* My solution rounds the adjusted counts to the nearest integer for clarity of presentation, but this is not a requirement. It might be better to keep 1 decimal place because the restated losses should then be more accurate.*

Step 1a calculate CDR as:  $CPC / UC$  (*UC = Ultimate Counts*)

## Reserving-D: PBS Method (Solution)

| AY      | Claims Disposal Rate (CDR) |       |       |       |
|---------|----------------------------|-------|-------|-------|
|         | 12                         | 24    | 36    | 48    |
| 2022    | 0.757                      | 0.973 | 0.991 | 1.000 |
| 2023    | 0.738                      | 0.958 | 0.990 |       |
| 2024    | 0.750                      | 0.980 |       |       |
| 2025    | 0.800                      |       |       |       |
| last CY | 0.800                      | 0.980 | 0.990 | 1.000 |

We assume the latest diagonal reflects current disposal rates. (This is the most recent CY.)  
We will restate all AYs to be at the level of this most recent CY.

<-- select LATEST DIAGONAL ( not all-period-average)

Step 1b restate CPC using CDRs from last CY

This "range" table is just a visual aid

| AY   | Restated Cumulative Paid Counts (CPC) |       |       |       |
|------|---------------------------------------|-------|-------|-------|
|      | 12                                    | 24    | 36    | 48    |
| 2025 | 3,504                                 | 4,292 | 4,336 | 4,380 |
| 2026 | 3,677                                 | 4,504 | 4,550 |       |
| 2027 | 3,563                                 | 4,365 |       |       |
| 2028 | 3,607                                 |       |       |       |

====>

| AY   | Range Relative to Original Paid Counts |       |       |       |
|------|----------------------------------------|-------|-------|-------|
|      | 12                                     | 24    | 36    | 48    |
| 2022 | 12-24                                  | 24-36 | 24-36 | 48-60 |
| 2023 | 12-24                                  | 24-36 | 36-48 |       |
| 2024 | 12-24                                  | 24-36 |       |       |
| 2025 | 12-24                                  |       |       |       |

Ex:  $3,504 = 0.8 \times 4380$   
Ex:  $4,504 = 0.98 \times 4596$

The range table is helpful in visualizing which data points or which regression parameters to use in Step 2.

Step 2 restate CPL using linear interpolation

or... restate CPL using exponential regression interpolation

| AY   | Restated Cumulative Paid Loss (CPL) |        |        |        |
|------|-------------------------------------|--------|--------|--------|
|      | 12                                  | 24     | 36     | 48     |
| 2022 | 8,946                               | 14,404 | 15,422 | 16,484 |
| 2023 | 10,140                              | 15,790 | 16,824 |        |
| 2024 | 9,140                               | 13,928 |        |        |
| 2025 | 9,113                               |        |        |        |

| AY   | Restated Cumulative Paid Loss (CPL) |        |        |        |
|------|-------------------------------------|--------|--------|--------|
|      | 12                                  | 24     | 36     | 48     |
| 2022 | 8,751                               | 14,719 | 15,785 | 16,484 |
| 2023 | 10,105                              | 15,606 | 16,824 |        |
| 2024 | 8,730                               | 13,928 |        |        |
| 2025 | 9,113                               |        |        |        |

Exs:

$8,946 = (3504-3314)/(4260-3314) \times (13664-7760) + 7760$   
 $15,790 = (4504-4404)/(4550-4404) \times (16824-13543) + 13543$   
 $15,422 = (4336-4260)/(4340-4260) \times (15515-13664) + 13664$

$8,751 = 1069 \times \text{EXP}(0.0006 \times 3504)$   
 $15,606 = 19 \times \text{EXP}(0.00149 \times 4504)$   
 $15,785 = 16 \times \text{EXP}(0.00159 \times 4336)$

Step 3a apply development method to restated CPL (Linear)

apply development method to restated CPL (Exp)

| LDFs | AY       | 12-24 | 24-36 | 36-48 | 48-ult |
|------|----------|-------|-------|-------|--------|
|      | 2022     | 1.610 | 1.071 | 1.069 |        |
|      | 2023     | 1.557 | 1.065 |       |        |
|      | 2024     | 1.524 |       |       |        |
|      | 2025     |       |       |       |        |
|      | selected | 1.564 | 1.068 | 1.069 | 1.034  |

or...

| LDFs | AY       | 12-24 | 24-36 | 36-48 | 48-ult |
|------|----------|-------|-------|-------|--------|
|      | 2022     | 1.682 | 1.072 | 1.044 |        |
|      | 2023     | 1.544 | 1.078 |       |        |
|      | 2024     | 1.595 |       |       |        |
|      | 2025     |       |       |       |        |
|      | selected | 1.607 | 1.075 | 1.044 | 1.022  |

| CDFs |            | 12-24 | 24-36 | 36-48 | 48-ult |
|------|------------|-------|-------|-------|--------|
|      | age -> ult | 1.846 | 1.180 | 1.105 | 1.034  |

| CDFs |            | 12-24 | 24-36 | 36-48 | 48-ult |
|------|------------|-------|-------|-------|--------|
|      | age -> ult | 1.843 | 1.147 | 1.067 | 1.022  |

| ULTs |          | '25@12 | '24@24 | '23@36 | '22@48 |
|------|----------|--------|--------|--------|--------|
|      | diagonal | 9,113  | 13,928 | 16,824 | 16,484 |
|      | ultimate | 16,821 | 16,441 | 18,595 | 17,043 |

| ULTs |          | '25@12 | '24@24 | '23@36 | '22@48 |
|------|----------|--------|--------|--------|--------|
|      | diagonal | 9,113  | 13,928 | 16,824 | 16,484 |
|      | ultimate | 16,793 | 15,972 | 17,947 | 16,843 |

Step 3b ultimate for AY 2025 = 16,821 <-- estimate based on LINEAR INTERPOLATION

ultimate for AY 2025 = 16,793 <-- estimate based on EXPONENTIAL REGRESSION INTERPOLATION

Step 3c The estimate of ultimate using exponential regression differs from the estimate using liner interpolation by:  
(Step 3c was just for fun to see if there is any real difference between linear and exponential interpolation.)

-0.17%

**Reading:** Friedland 15 (Evaluating Methods)  
**Model:** Emergence Patterns  
**Problem Type:** Calculate emergence in a future period

## Reserving-D: Emergence (Problem)

**Find** Calculate the emergence of paid claims in the period 39 to 42

**Given** Here is the current information for a particular AY:

|      |        |    |    |        |     |
|------|--------|----|----|--------|-----|
| paid | claims | at | 36 | months | 370 |
|------|--------|----|----|--------|-----|

|               |     |      |
|---------------|-----|------|
| 12 - ultimate | CDF | 2.50 |
| 24 - ultimate | CDF | 1.31 |
| 36 - ultimate | CDF | 1.13 |
| 48 - ultimate | CDF | 1.06 |
| 60 - ultimate | CDF | 1.03 |
| 72 - ultimate | CDF | 1.01 |

**Step 1** determine the 12-month period containing the period you're given

period 39 to 42 is contained in period 36 to 48

**Step 2** calculate the quantities required in the emergence formula (for step 3)

$$\begin{aligned}\text{ultimate} &= \text{paid @36} \times 36\text{-ultimate CDF} \\ &= 370 \times 1.13 \\ &= 418\end{aligned}$$

$$\begin{aligned}\text{unpaid @ 36} &= \text{ultimate} - \text{paid @36} \\ &= 418 - 370 \\ &= 48\end{aligned}$$

$$\begin{aligned}\% \text{paid @ 36} &= 1 / 36\text{-ultimate CDF} \\ &= 1 / 1.13 \\ &= 88.5\%\end{aligned}$$

$$\begin{aligned}\% \text{paid @ 48} &= 1 / 48\text{-ultimate CDF} \\ &= 1 / 1.06 \\ &= 94.3\%\end{aligned}$$

$$\begin{aligned}\% \text{unpaid @ 36} &= 1 - \% \text{paid @ 36} \\ &= 1 - 88.5\% \\ &= 11.5\%\end{aligned}$$

**Step 3** calculate the emergence in the 12-month period using the results from step 2

$$\begin{aligned}12\text{-mth emergence} &= \text{unpaid @ 36} \times (\% \text{paid @ 48} - \% \text{paid @ 36}) / (\% \text{unpaid @ 36}) \\ &= 48 \times (94.3\% - 88.5\%) / 11.5\% \\ &= 24.4\end{aligned}$$

**Step 4** if the given period was NOT a 12-month interval then you must interpolate to get the final answer

given period =====> 39 to 42 which is 3 months

$$\begin{aligned}3\text{-mth emergence} &= 24.4 \times 3 / 12 \\ &= 6.1 \quad \text{<== final answer}\end{aligned}$$

**Reading:** Friedland 14 (S/S)  
**Model:** 2016.Fall #24(b)  
**Problem Type:** Ratio method (additive/multiplicative) for S/S

## Reserving-E: SS Ratio Method (Problem)

**Problem** Use the **multiplicative** ratio method to estimate the **ultimate** S/S for AY **2014**

cumulative paid claims:

| AY   | 12  | 24  | 36  | 48  |
|------|-----|-----|-----|-----|
| 2011 | 200 | 560 | 570 | 570 |
| 2012 | 150 | 250 | 400 |     |
| 2013 | 150 | 350 |     |     |
| 2014 | 50  |     |     |     |

cumulative paid SS:

| AY   | 12 | 24 | 36  | 48  |
|------|----|----|-----|-----|
| 2011 | 20 | 90 | 100 | 100 |
| 2012 | 16 | 40 | 70  |     |
| 2013 | 15 | 56 |     |     |
| 2014 | 5  |    |     |     |

selected ultimate claims by AY (using paid claim development)

| AY   | ult clms |                                                |
|------|----------|------------------------------------------------|
| 2011 | 570      | <== paid claims development method             |
| 2012 | 400      | <== paid claims development method             |
| 2013 | 458      | <== paid claims development method             |
| 2014 | 150      | <== given in the statement of the exam problem |

**When selecting development factors, you may select a SIMPLE AVERAGE in this problem.**

\* There is no development beyond 48 months.

## Reserving-E: SS Ratio Method (Solution)

**Step 1:** Ratio of *cumulative* paid S/S to *cumulative* paid claims:

| AY   | 12     | 24     | 36     | 48     |
|------|--------|--------|--------|--------|
| 2011 | 10.00% | 16.07% | 17.54% | 17.54% |
| 2012 | 10.67% | 16.00% | 17.50% |        |
| 2013 | 10.00% | 16.00% |        |        |
| 2014 | 10.00% |        |        |        |

**Step 2:** Development triangle (*multiplicative*):

| AY        | 12-24  | 24-36  | 36-48  | 48-    |                                          |
|-----------|--------|--------|--------|--------|------------------------------------------|
| 2011      | 1.6070 | 1.0915 | 1.0000 |        | <=== MULTIPLICATIVE development          |
| 2012      | 1.4995 | 1.0938 |        |        |                                          |
| 2013      | 1.6000 |        |        |        |                                          |
| 2014      |        |        |        |        |                                          |
| selected  | 1.5688 | 1.0927 | 1.0000 | 1.0000 | <=== tail = 1.0                          |
| cum       | 1.7142 | 1.0927 | 1.0000 | 1.0000 | <=== selected x (previous cum)           |
| Ult Ratio | 17.14% | 17.48% | 17.50% | 17.54% | <=== cum x (latest diagonal from Step 1) |

  

| AY       | 2014 | 2013 | 2012 | 2011 |                                                           |
|----------|------|------|------|------|-----------------------------------------------------------|
| ult clms | 150  | 458  | 400  | 570  | <=== given information<br>(repeated here for convenience) |
| pd S/S   | 5    | 56   | 70   | 100  |                                                           |

**Step 3:** Ultimate S/S = (Ultimate Ratio) x (Ultimate Claims)  
 Unpaid S/S = (Ultimate S/S) - (Paid S/S)

| AY   | Ult S/S | UnPd S/S |
|------|---------|----------|
| 2011 | 100.0   | (0.0)    |
| 2012 | 70.0    | -        |
| 2013 | 80.1    | 24.1     |
| 2014 | 25.7    | 20.7     |
|      | 275.7   | 44.7     |

**Step 4:** ultimate S/S for AY 2014 = 25.7 <=== final answer

For **COMPARISON**, the S/S ultimate and unpaid values using standard paid S/S development are:

Development method:

% Difference: [ (ratio method) - (development method) ] / (ratio method)

| AY   | Ult S/S | UnPd S/S |
|------|---------|----------|
| 2011 | 100.0   | -        |
| 2012 | 70.0    | -        |
| 2013 | 80.1    | 24.1     |
| 2014 | 25.6    | 20.6     |
|      | 275.7   | 44.7     |

| Ult S/S | UnPd S/S |
|---------|----------|
| 0%      | -        |
| 0%      | -        |
| 0%      | 0%       |
| 0%      | 1%       |
| 0%      | 0%       |



**Reading:** Friedland 14 (Reinsurance)  
**Model:** 2017.Fall #25  
**Problem Type:** Apply reinsurance treaties and calculate net unpaid claims

## Reserving-E: Reinsurance (Problem)

**Problem** Calculate the net unpaid claims for all accident years using reported claims development.

cumulative gross reported claims:

(no reported claim development beyond 48 months)

| AY   | 12    | 24    | 36    | 48    |
|------|-------|-------|-------|-------|
| 2013 | 2,757 | 5,570 | 6,880 | 7,047 |
| 2014 | 2,345 | 4,104 | 5,121 |       |
| 2015 | 2,639 | 4,677 |       |       |
| 2016 | 2,802 |       |       |       |

cumulative ceded reported claims:

(under excess-of-loss treaty)

| AY   | 12  | 24  | 36    | 48    |
|------|-----|-----|-------|-------|
| 2013 | 0   | 745 | 1,332 | 1,332 |
| 2014 | 0   | 0   | 402   |       |
| 2015 | 154 | 328 |       |       |
| 2016 | 0   |     |       |       |

cumulative net paid claims:

(net of excess-of-loss treaty)

| AY   | pd clm |
|------|--------|
| 2013 | 5,102  |
| 2014 | 3,834  |
| 2015 | 2,840  |
| 2016 | 1,385  |

stop-loss limits

(applies AFTER excess-of-loss treaty)

| AY   | limit |
|------|-------|
| 2013 | 5,000 |
| 2014 | 5,000 |
| 2015 | 5,000 |
| 2016 | none  |

*There is no further loss development past 48 months*

## Reserving-E: Reinsurance (Solution)

**Step 1:** Calculate the net reported claims triangle as (gross reported) - (ceded reported)

| AY   | 12    | 24    | 36    | 48    |
|------|-------|-------|-------|-------|
| 2013 | 2,757 | 4,825 | 5,548 | 5,715 |
| 2014 | 2,345 | 4,104 | 4,719 |       |
| 2015 | 2,485 | 4,349 |       |       |
| 2016 | 2,802 |       |       |       |

**Step 2:** Calculate net ultimate claims using reported development on the net triangle above (net of excess-of-loss)

| AY                     | 12-24  | 24-36  | 36-48  | 48-    |      |                                     |
|------------------------|--------|--------|--------|--------|------|-------------------------------------|
| 2013                   | 1.7501 | 1.1498 | 1.0301 |        | <=== | MULTIPLICATIVE development          |
| 2014                   | 1.7501 | 1.1499 |        |        |      |                                     |
| 2015                   | 1.7501 |        |        |        |      |                                     |
| 2016                   |        |        |        |        |      |                                     |
| selected               | 1.7501 | 1.1499 | 1.0301 | 1.0000 | <=== | tail = 1.0                          |
| cum                    | 2.0730 | 1.1845 | 1.0301 | 1.0000 | <=== | selected x (previous cum)           |
| Ult Clms<br>net of EoL | 5,809  | 5,151  | 4,861  | 5,715  | <=== | cum x (latest diagonal from Step 1) |

**Step 3:** Put the results from Step 2 into a table and apply the given stop-loss limits

| AY    | Ult Clms<br>net of EoL | stop-loss<br>limit | Ult Clms<br>net of ALL | net<br>paid | net<br>unpaid |      |              |
|-------|------------------------|--------------------|------------------------|-------------|---------------|------|--------------|
| 2013  | 5,715                  | 5,000              | 5,000                  | 5,102       | 0             | <=== | final answer |
| 2014  | 4,861                  | 5,000              | 4,861                  | 3,834       | 1,027         | <=== | final answer |
| 2015  | 5,151                  | 5,000              | 5,000                  | 2,840       | 2,160         | <=== | final answer |
| 2016  | 5,809                  | none               | 5,809                  | 1,385       | 4,424         | <=== | final answer |
| TOTAL |                        |                    |                        |             | 7,611         |      |              |

**Reading:** Friedland 16 (ALAE)  
**Model:** 2019.Spring #24  
**Problem Type:** Additive or Multiplicative approach for ALAE

## Reserving-E: ALAE - Multiplicative (Problem)

**Problem** Use the **multiplicative** approach to estimate the **ultimate** ALAE for AY **2018**

cumulative paid claims:

| AY   | 12    | 24     | 36     | 48     |
|------|-------|--------|--------|--------|
| 2015 | 3,800 | 10,640 | 15,960 | 17,556 |
| 2016 | 3,900 | 10,920 | 15,600 |        |
| 2017 | 3,850 | 11,858 |        |        |
| 2018 | 4,050 |        |        |        |

cumulative paid ALAE:

| AY   | 12 | 24  | 36  | 48  |
|------|----|-----|-----|-----|
| 2015 | 77 | 316 | 512 | 571 |
| 2016 | 81 | 337 | 517 |     |
| 2017 | 75 | 334 |     |     |
| 2018 | 82 |     |     |     |

selected ultimate claims by AY

| AY   | ult clms | <=== | sometimes you are not given the ultimate claims<br>- you would then have to calculate them using an appropriate method<br>- see <a href="#">2016.Spring #23</a> |
|------|----------|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2015 | 17,500   |      |                                                                                                                                                                 |
| 2016 | 17,900   |      |                                                                                                                                                                 |
| 2017 | 17,600   |      |                                                                                                                                                                 |
| 2018 | 18,500   |      |                                                                                                                                                                 |

**When selecting development factors, you may select a SIMPLE AVERAGE in this problem.**

## Reserving-E: ALAE - Multiplicative (Solution)

Step 1: Ratio of *cumulative* paid ALAE to *cumulative* paid claims:

| AY   | 12    | 24    | 36    | 48    |
|------|-------|-------|-------|-------|
| 2015 | 2.03% | 2.97% | 3.21% | 3.25% |
| 2016 | 2.08% | 3.09% | 3.31% |       |
| 2017 | 1.95% | 2.82% |       |       |
| 2018 | 2.02% |       |       |       |

Step 2: Development triangle (*either additive or multiplicative as appropriate*):

| AY        | 12-24  | 24-36  | 36-48  | 48-    |                                                          |
|-----------|--------|--------|--------|--------|----------------------------------------------------------|
| 2015      | 1.4631 | 1.0808 | 1.0125 |        | <=== MULTIPLICATIVE development                          |
| 2016      | 1.4856 | 1.0712 |        |        |                                                          |
| 2017      | 1.4462 |        |        |        |                                                          |
| 2018      |        |        |        |        |                                                          |
| selected  | 1.4650 | 1.0760 | 1.0125 | 1.0000 | <=== tail = 0.0 if additive OR 1.0 if multiplicative     |
| cum       | 1.5961 | 1.0895 | 1.0125 | 1.0000 | <=== MULTIPLICATIVE: selected x (previous cum)           |
| Ult Ratio | 3.22%  | 3.07%  | 3.35%  | 3.25%  | <=== MULTIPLICATIVE: cum x (latest diagonal from Step 1) |

  

| AY       | 2018   | 2017   | 2016   | 2015   |                                 |
|----------|--------|--------|--------|--------|---------------------------------|
| ult clms | 18,500 | 17,600 | 17,900 | 17,500 | <=== given information          |
| pd ALAE  | 82     | 334    | 517    | 571    | (repeated here for convenience) |

Step 3: Ultimate ALAE = (Ultimate Ratio) x (Ultimate Claims)  
 Unpad ALAE = (Ultimate ALAE) - (Paid ALAE)

| AY   | Ult ALAE | UnPd ALAE |
|------|----------|-----------|
| 2015 | 568.8    | (2.3)     |
| 2016 | 599.7    | 82.7      |
| 2017 | 540.3    | 206.3     |
| 2018 | 595.7    | 513.7     |
|      | 2,304.4  | 800.4     |

Step 4: ultimate ALAE for AY 2018 = 595.7 <=== final answer

**Reading:** Friedland 16 (ALAE)  
**Model:** 2019.Spring #24  
**Problem Type:** Additive or Multiplicative approach for ALAE

## Reserving-E: ALAE - Additive (Problem)

**Problem** Use the **additive** approach to estimate the **ultimate** ALAE for AY **2019**

cumulative paid claims:

| AY   | 12    | 24     | 36     | 48     |
|------|-------|--------|--------|--------|
| 2017 | 4,300 | 13,900 | 20,900 | 22,800 |
| 2018 | 4,500 | 13,900 | 21,500 |        |
| 2019 | 3,700 | 11,800 |        |        |
| 2020 | 4,400 |        |        |        |

cumulative paid ALAE:

| AY   | 12  | 24  | 36  | 48  |
|------|-----|-----|-----|-----|
| 2017 | 116 | 507 | 803 | 866 |
| 2018 | 99  | 428 | 719 |     |
| 2019 | 89  | 396 |     |     |
| 2020 | 110 |     |     |     |

selected ultimate claims by AY

| AY   | ult clms | <=== | sometimes you are not given the ultimate claims<br>- you would then have to calculate them using an appropriate method<br>- see 2016.Spring #23 |
|------|----------|------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| 2017 | 22,800   |      |                                                                                                                                                 |
| 2018 | 23,455   |      |                                                                                                                                                 |
| 2019 | 19,633   |      |                                                                                                                                                 |
| 2020 | 23,209   |      |                                                                                                                                                 |

**When selecting development factors, you may select a SIMPLE AVERAGE in this problem.**

## Reserving-E: ALAE - Additive (Solution)

Step 1: Ratio of *cumulative* paid ALAE to *cumulative* paid claims:

| AY   | 12    | 24    | 36    | 48    |
|------|-------|-------|-------|-------|
| 2017 | 2.70% | 3.65% | 3.84% | 3.80% |
| 2018 | 2.20% | 3.08% | 3.34% |       |
| 2019 | 2.41% | 3.36% |       |       |
| 2020 | 2.50% |       |       |       |

Step 2: Development triangle (*either additive or multiplicative as appropriate*):

| AY        | 12-24  | 24-36  | 36-48   | 48-    |      |                                                        |
|-----------|--------|--------|---------|--------|------|--------------------------------------------------------|
| 2017      | 0.0095 | 0.0019 | -0.0004 |        | <=== | ADDITIVE development                                   |
| 2018      | 0.0088 | 0.0026 |         |        |      |                                                        |
| 2019      | 0.0095 |        |         |        |      |                                                        |
| 2020      |        |        |         |        |      |                                                        |
| selected  | 0.0093 | 0.0023 | -0.0004 | 0.0000 | <=== | tail = 0.0 if additive <u>OR</u> 1.0 if multiplicative |
| cum       | 0.0112 | 0.0019 | -0.0004 | 0.0000 | <=== | ADDITIVE: selected + (previous cum)                    |
| Ult Ratio | 3.62%  | 3.55%  | 3.30%   | 3.80%  | <=== | ADDITIVE: cum + (latest diagonal from Step 1)          |

  

| AY       | 2020   | 2019   | 2018   | 2017   |      |                                 |
|----------|--------|--------|--------|--------|------|---------------------------------|
| ult clms | 23,209 | 19,633 | 23,455 | 22,800 | <=== | given information               |
| pd ALAE  | 110    | 396    | 719    | 866    |      | (repeated here for convenience) |

Step 3: Ultimate ALAE = (Ultimate Ratio) x (Ultimate Claims)  
 Unpad ALAE = (Ultimate ALAE) - (Paid ALAE)

| AY   | Ult ALAE | UnPd ALAE |
|------|----------|-----------|
| 2017 | 866.4    | 0.4       |
| 2018 | 774.0    | 55.0      |
| 2019 | 697.0    | 301.0     |
| 2020 | 840.2    | 730.2     |
|      | 3,177.6  | 1,086.6   |

Step 4: ultimate ALAE for AY 2019 = 697.0 <=== final answer

**Reading:** Friedland 17 (ULAE)  
**Model:** 2018.Spring #23  
**Problem Type:** Classical/Kittel approach for ULAE

## Reserving-E: ULAE - Classical (Problem)

**Problem** Use the **classical** approach to estimate the **unpaid** ULAE for AY **2022**  
 occurrence <= policy type

| CY   | paid<br>ULAE | paid<br>claims | incurred<br>claims | <== incurred <u>includes</u> reported & IBNR |
|------|--------------|----------------|--------------------|----------------------------------------------|
| 2018 | 0            | 0              | 0                  |                                              |
| 2019 | 52,000       | 350,000        | 530,000            |                                              |
| 2020 | 50,000       | 415,000        | 490,000            |                                              |
| 2021 | 51,000       | 496,000        | 450,000            |                                              |
| 2022 | 56,000       | 384,000        | 490,000            |                                              |

|     |                                                                       |
|-----|-----------------------------------------------------------------------|
| 700 | case outstanding (latest AY)                                          |
| 400 | total IBNR (latest AY)                                                |
| 60% | % of total IBNR attributed to future case development on known claims |

|     |                                                                       |
|-----|-----------------------------------------------------------------------|
| 60% | percent of unallocated work that occurs when a claim is <b>opened</b> |
| 40% | percent of unallocated work that occurs when a claim is <b>closed</b> |

Step 1: classical ULAE ratio ==> (paid ULAE) / (paid claims)

| CY   | paid<br>ULAE | paid<br>claims | ULAE<br>ratio |
|------|--------------|----------------|---------------|
| 2018 | 0            | 0              |               |
| 2019 | 44,000       | 332,000        | 13.3%         |
| 2020 | 54,000       | 365,000        | 14.8%         |
| 2021 | 47,000       | 466,000        | 10.1%         |
| 2022 | 53,000       | 333,000        | 15.9%         |

Sometimes there is a **trend** in ULAE ratios.  
If so, you may need to use **judgement**  
instead of just selecting the average

10.0% <== selected (average)

Step 2: apply formula for unpaid ULAE

$$\text{unpaid ULAE} = (\text{ULAE ratio}) \times [40\% \times (\text{Case} + \text{IBNER})] + 100\% \times \text{IBNYR}$$

**where:**

$$\begin{aligned} \text{IBNER} &= 60\% \times \text{Total IBNR} &<== \text{Incurred But Not ENOUGH Reported} \\ &= 60\% \times 400 \\ &= 240 \end{aligned}$$

$$\begin{aligned} \text{IBNYR} &= \text{Tot IBNR} - \text{IBNER} &<== \text{Incurred But Not YET Reported} \\ &= 400 - 240 \\ &= 160 \end{aligned}$$

**therefore:**

$$\begin{aligned} \text{unpaid ULAE} &= 10.0\% \times [40\% \times 940] + 100\% \times 160 \\ \text{unpaid ULAE} &= 54 &<== \text{final answer} \end{aligned}$$



**Reading:** Friedland 17 (ULAE)  
**Model:** 2019.Fall #24  
**Problem Type:** Kittel approach for ULAE

## Reserving-E: ULAE - Kittel (Problem)

**Problem** Use the **Kittel** approach to estimate the **unpaid** ULAE at CY **2024** year-end

55% <== Expected Clams Ratio (ECR)  
 claims-made <== policy type

| CY   | paid<br>claims | incurred<br>claims | paid<br>ULAE |
|------|----------------|--------------------|--------------|
| 2021 | 16,591         | 32,700             | 1,991        |
| 2022 | 16,400         | 35,800             | 1,825        |
| 2023 | 18,100         | 34,500             | 1,825        |
| 2024 | 17,100         | 32,400             | 1,825        |

| Report<br>Year | earned<br>premium | paid<br>claims | reported<br>claims | %<br>unrpt'd |
|----------------|-------------------|----------------|--------------------|--------------|
| 2021           | 68,000            | 20,900         | 28,700             | 12.0%        |
| 2022           | 65,900            | 12,300         | 23,500             | 21.6%        |
| 2023           | 66,700            | 7,300          | 19,000             | 36.7%        |
| 2024           | 64,000            | 4,200          | 17,100             | 68.4%        |

**Step 1:** Kittel ULAE ratio ==> (paid ULAE) / AVG [ (paid claims) , (incurred claims) ]

| CY   | paid ULAE | average (pd, inc) | ULAE ratio |
|------|-----------|-------------------|------------|
| 2021 | 1,991     | 24,646            | 8.1%       |
| 2022 | 1,825     | 26,100            | 7.0%       |
| 2023 | 1,825     | 26,300            | 6.9%       |
| 2024 | 1,825     | 24,750            | 7.4%       |

Sometimes there is a **trend** in ULAE ratios.  
If so, you may need to use **judgement** instead of just selecting the average

7.4% <== selected (average)

**Step 2a** calculate ultimate (use Bornhuetter-Ferguson reported method)  
(usually, we are given case O/S and IBNR in these types of problems but here we have to calculate it ourselves)

| AY   | ultimate | = | reported | + | %unrptd | x | ECR | x | EP     |
|------|----------|---|----------|---|---------|---|-----|---|--------|
| 2021 | 33,188   | = | 28,700   | + | 12.0%   | x | 55% | x | 68,000 |
| 2022 | 31,329   | = | 23,500   | + | 21.6%   | x | 55% | x | 65,900 |
| 2023 | 32,463   | = | 19,000   | + | 36.7%   | x | 55% | x | 66,700 |
| 2024 | 41,177   | = | 17,100   | + | 68.4%   | x | 55% | x | 64,000 |
|      | 138,157  |   | 88,300   |   |         |   |     |   |        |

**Step 2b** calculate case O/S and IBNR in total for all years

|          |   |          |   |          |   |         |   |        |   |               |
|----------|---|----------|---|----------|---|---------|---|--------|---|---------------|
| case O/S | = | reported | - | paid     | = | 88,300  | - | 44,700 | = | <u>43,600</u> |
| IBNR     | = | ultimate | - | reported | = | 138,157 | - | 88,300 | = | <u>49,857</u> |

**Trick:** Since these policies are **claims-made**, there is no pure IBNR (no IBNYR).

|            |   |       |   |       |
|------------|---|-------|---|-------|
| total IBNR | = | IBNER | + | IBNYR |
| 49,857     | = | IBNER | + | 0     |

Therefore IBNER = 49,857

**Step 3** apply formula for unpaid ULAE

$$\begin{aligned}
 \text{unpd ULAE} &= \text{ULAE ratio} \times [ 50\% \times ( \text{case} + \text{IBNER} ) + 100\% \times \text{IBNYR} ] \\
 &= 7.4\% \times [ 50\% \times ( 43,600 + 49,857 ) + 100\% \times 0 ] \\
 &= 7.4\% \times 46,729 \\
 &= 3,458 \\
 &\quad \text{(final answer)}
 \end{aligned}$$

**Note:** Part (b) of this exam problem asked how the calculations would change if the policies were "occurrence" instead of claims-made.

The answer is that "occurrence" policies have pure IBNR (IBNYR). You could use the same formula as in Step 3, but you would need a way of separating the total IBNR into its components IBNER and IBNYR.

**Reading:** Friedland 17 (ULAE)  
**Model:** ULAE Example  
**Problem Type:** Congor & Nolibos approach for ULAE

## Reserving-E: ULAE - Conger-Nolibos (Problem)

**Find** Estimate the unpaid ULAE as of the latest given year-end using all 3 versions of the Congor and Nolibos approach.

**Given** independent estimate of ultimate claims for all AYs 460,000

|                                       |     |
|---------------------------------------|-----|
| % of ULAE spend on opening claims     | 55% |
| % of ULAE spend on maintaining claims | 30% |
| % of ULAE spend on closing claims     | 15% |

| CY   | paid<br>ULAE<br>in CY<br>M | ultimate<br>for clms<br>reported<br>in CY<br>R | paid<br>loss<br>in CY<br>P | ultimate<br>for clms<br>closed<br>in CY<br>C |
|------|----------------------------|------------------------------------------------|----------------------------|----------------------------------------------|
| 2021 | 5,000                      | 57,500                                         | 22,500                     | 21,500                                       |
| 2022 | 5,000                      | 70,200                                         | 28,400                     | 28,000                                       |
| 2023 | 5,500                      | 84,900                                         | 34,600                     | 35,600                                       |
| 2024 | 5,500                      | 106,100                                        | 43,300                     | 44,100                                       |
| 2025 | 5,900                      | 131,600                                        | 52,000                     | 57,300                                       |

## Reserving-E: ULAE - Conger-Nolibos (Solution)

Step 1a calculate the claims basis B

| CY   | 55% | x | R       | + | 30% | x | P       | + | 15% | x | C       | claims basis B |
|------|-----|---|---------|---|-----|---|---------|---|-----|---|---------|----------------|
| 2021 | 55% | x | 57,500  | + | 30% | x | 22,500  | + | 15% | x | 21,500  | 41,600         |
| 2022 | 55% | x | 70,200  | + | 30% | x | 28,400  | + | 15% | x | 28,000  | 51,330         |
| 2023 | 55% | x | 84,900  | + | 30% | x | 34,600  | + | 15% | x | 35,600  | 62,415         |
| 2024 | 55% | x | 106,100 | + | 30% | x | 43,300  | + | 15% | x | 44,100  | 77,960         |
| 2025 | 55% | x | 131,600 | + | 30% | x | 52,000  | + | 15% | x | 57,300  | 96,575         |
|      |     |   | 450,300 |   |     |   | 180,800 |   |     |   | 186,500 | 329,880        |

Step 1b select a ULAE ratio W:  $W = M / B$ 

| CY   | paid ULAE M | claims basis B | ULAE ratio W |
|------|-------------|----------------|--------------|
| 2021 | 5,000       | 41,600         | 12.0%        |
| 2022 | 5,000       | 51,330         | 9.7%         |
| 2023 | 5,500       | 62,415         | 8.8%         |
| 2024 | 5,500       | 77,960         | 7.1%         |
| 2025 | 5,900       | 96,575         | 6.1%         |
|      | 26,900      | 329,880        | 8.154%       |

If you decided at the beginning you are going to use a weighted average for B then you only need the totals line in Step 1a and 1b. The problem with that however is that you may not see trends in the ULAE ratio.

&lt;===== you can select this weighted average or choose another reasonable ratio

Step 2a calculate the estimate of unpaid ULAE using the Expected Claims approach

$$\begin{aligned}
 \text{unpd ULAE} &= W \times L - M \\
 &= 8.154\% \times 460,000 - 26,900 \\
 &= 10,608
 \end{aligned}$$

Step 2b calculate the estimate of unpaid ULAE using the Bornhuetter-Ferguson approach

$$\begin{aligned}
 \text{unpd ULAE} &= W \times (L - B) \\
 &= 8.154\% \times (460,000 - 329,880) \\
 &= 10,610
 \end{aligned}$$

Step 2b calculate the estimate of unpaid ULAE using the Development approach

$$\begin{aligned}
 \text{unpd ULAE} &= M \times (L / B - 1) \\
 &= 26,900 \times (460,000 / 329,880 - 1) \\
 &= 10,611
 \end{aligned}$$

**Notes:** You get the same answer for all 3 methods if your selection for W is the weighted average. You also have to keep enough decimal places or they may differ due to rounding.

If you choose something other than the weighted average, which could be entirely reasonable depending on the circumstances then you won't in general get the same answer.