PowerPack Station 2: One-Stop PDF

RESERVING

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Reserving-A: Information in Triangles (Problem)

Find

(a)	cumulative paid loss for AY	2023	@	36	months
(b)	incremental paid loss for AY	2023	@	12	months
(c)	cumulative reported loss for AY	2022	@	36	months
(d)	incremental reported loss for AY	2022	@	48	months
(e)	cumulative reported count for AY	2022	@	48	months
(f)	total paid loss for CY	2023			
(g)	total rptd loss for CY	2021			

Given

Amounts	Paid Loss	Development Age				
AY	**12**	**24**	**36**	**48**	**60**	**72**
2020	2,160	5,040	7,200	9,360	10,080	11,520
2021	1,217	3,650	6,084	7,301	7,909	
2022	1,080	2,700	4,320	5,400		
2023	469	1,406	2,343			
2024	399	1,197				
2025	950					

Amounts	O/S Loss	Development Age				
AY	**12**	**24**	**36**	**48**	**60**	**72**
2020	3,600	2,880	2,160	1,440	1,440	0
2021	3,650	3,042	1,825	1,217	1,217	
2022	2,160	1,620	1,080	540		
2023	1,406	1,406	937			
2024	1,197	1,197				
2025	950					

<== *same as Case O/S*

Amounts	Paid Count	Development Age				
AY	**12**	**24**	**36**	**48**	**60**	**72**
2020	3	7	10	13	14	16
2021	2	6	10	12	13	
2022	2	5	8	10		
2023	1	3	5			
2024	1	3				
2025	2					

Amounts	O/S Count	Development Age				
AY	**12**	**24**	**36**	**48**	**60**	**72**
2020	5	4	3	2	2	0
2021	6	5	3	2	2	
2022	4	3	2	1		
2023	3	3	2			
2024	3	3				
2025	2					

10 4 11 8 8 8 8 8 8 8 8 8 6

(a)	2,343	<==	*directly from paid loss triangle*						
(b)	469	=	469	-	0				
(c)	5,400	=	4,320	+	1,080				
(d)	540	=	5,400	+	540	-	4,320	-	1,080
(e)	11	=	10	+	1				
(f)	6,682	<==	*directly from alternate format of paid loss triangle below*						
(g)	7,027	<==	*directly from alternate format of reported loss triangle below*						

side calc for CY loss: express triangles in alternate format

Amounts	Paid Loss		as of Calendar Year End			
AY	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**
2020	2,160	5,040	7,200	9,360	10,080	11,520
2021		1,217	3,650	6,084	7,301	7,909
2022			1,080	2,700	4,320	5,400
2023				469	1,406	2,343
2024					399	1,197
2025						950
	2,160	6,257	11,930	18,613	23,506	29,320
CY paid	2,160	4,097	5,674	6,682	4,893	5,814
CY	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**

Amounts	Rptd Loss		as of Calendar Year End			
AY	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**
2020	5,760	7,920	9,360	10,800	11,520	11,520
2021		4,867	6,692	7,909	8,518	9,126
2022			3,240	4,320	5,400	5,940
2023				1,874	2,812	3,280
2024					1,596	2,394
2025						1,901
	5,760	12,787	19,292	24,904	29,845	34,161
CY rptd	5,760	7,027	6,505	5,611	4,942	4,316
CY	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**

Reading: Friedland 05 (Triangles)
Model: 2014.Fall #14
Problem Type: Building Triangles from Raw Claims Data

Find Build the **AY (Accident Year)** paid & reported claim triangles. (Unless stated otherwise these are cumulative triangles)

Given

			2020		2021		2022	
				ending		ending		ending
			paid in	case O/S	paid in	case O/S	paid in	case O/S
claim id	AY	RY	CY	CY	CY	CY	CY	CY
1	2020	2020	600	0	0	0	0	0
2	2020	2020	700	0	0	0	0	0
3	2020	2021	-	-	900	300	400	0
4	2020	2022	-	-	-	-	600	0
5	2020	2022	-	-	-	-	600	100
6	2021	2021	-	-	300	0	0	0
7	2021	2021	-	-	500	400	500	200
8	2021	2021	-	-	800	400	400	200
9	2021	2021	-	-	600	200	400	200
10	2022	2022	-	-	-	-	700	100
11	2022	2023	-	-	-	-	-	-
12	2022	2023	-	-	-	-	-	-
13	2022	2024	-	-	-	-	-	-
14	2022	2024	-	-	-	-	-	-
15	-	-	-	-	-	-	-	-
16	-	-	-	-	-	-	-	-
17	-	-	-	-	-	-	-	-
18	-	-	-	-	-	-	-	-

Step 1 Sum across **AY** (This is simple in Excel but easy to make a mistake with pencil and paper.)

Suggestion: *Use a ruler to draw a horizontal line between one AY and the next.*

		2020		2021		2022	
		paid in	ending case O/S	paid in	ending case O/S	paid in	ending case O/S
AY	RY	CY	CY	CY	CY	CY	CY
2020	all	1,300	0	900	300	1,600	100
2021	all	0	0	2,200	1,000	1,300	600
2022	all	0	0	0	0	700	100
2023	all	0	0	0	0	0	0

Step 2 Transfer the colour-coded values from Step 1 into these triangles…

incremental PAID claims

AY	12	24	36
2020	1,300	900	1,600
2021	2,200	1,300	
2022	700		
2023			

case O/S

AY	12	24	36
2020	0	300	100
2021	1,000	600	
2022	100		
2023			

Step 3 (final answer) Form the CUMULATIVE paid triangle by summing across rows in the incremental paid triangle

cumulative PAID claims

AY	12	24	36
2020	1,300	2,200	3,800
2021	2,200	3,500	
2022	700		
2023			

Ex: 2,200 = 1,300 + 900

3,800 = 1,300 + 900 + 1,600

Step 4 (final answer) Form the CUMULATIVE reported triangle: *cumulative reported claims = (cumulative paid claims) + (Case O/S)*

cumulative REPORTED claims

AY	12	24	36
2020	1,300	2,500	3,900
2021	3,200	4,100	
2022	800		
2023			

Reading: Friedland 07 (Development Method)
Model: Simple Example of Development Technique
Problem Type: Paid Claims Development

Find Calculate ultimate claims for all accident years using data as of year-end 2023

Given **cumulative paid claims**

AY	12	24	36	48
2020	1,590	1,591	1,655	1,671
2021	1,606	1,792	1,840	
2022	1,605	1,700		
2023	1,604			

Assume no development past 48 months. In other words, the triangle is fully dveloped by 48 months.

The paid claims on the latest diagonal are in brown font for instructional purposes within the solution.

Step A =====> **link ratios for paid claim triangle**

AY	12-24	24-36	36-48	48-ult
2020	1.001	1.040	1.010	
2021	1.116	1.027		
2022	1.059			
2023				
Step B ===== **selected**	1.059	1.034	1.010	**1.050**

Tail Factor: The triangle is fully developed as of 48 months. That means the 48-ult tail factor is equal to 1.0

Step C =====> **calculate age-to-ultimate LDFs**

	12-ult	24-ult	36-ult	48-ult
age -> ult	1.160	1.096	1.060	**1.050**

<======= (**selected**) x (prior [age -> ult])

(calculate from right-to-left)

Step D =====> **calculate ultimate losses based on latest paid losses**

	'23@12	**'22@24**	**'21@36**	**'20@48**
diagonal	1,604	1,700	1,840	1,671
final answers ===> ultimate	1,860	1,863	1,951	1,755

<======= (**diagonal**) x (age -> ult)

Sometimes it's nice to present the ultimates in a column to the right of the original triangle:

cumulative paid claims

AY	12	24	36	48
2020	1,590	1,591	1,655	1,671
2021	1,606	1,792	1,840	
2022	1,605	1,700		
2023	1,604			

estimated ultimates	real ultimates	% error
1,755	240	631%
1,951	240	713%
1,863	240	676%
1,860	240	675%

Interesting side note:

*This example was created using my simulation software **SimPolicy**. One of the input parameters to the simulation is the value of the ultimate loss. For this simulation, each AY was given the same **ultimate loss of 240**. That means we can see how accurate our estimates are. More to the point, we can often see how **inaccurate** our estimates are regardless of how we select our LDFs (Loss Development Factors) in Step B.*

Moral:

Don't agonize for too long over selecting LDFs. In a real-life situation there will be a lot of noise or random variation that cannot be accounted for in any reserving method. Do the best you can with the information you've got but make allowances for the fact that your estimates will never be exactly right, especially for AYs at early stages of development.

Reading: Friedland 09 (Bornhuetter-Ferguson Method)
Model: BF Method
Problem Type: Reserving Methods - Simple Example of BF

Find Calculate the ultimate for AY 2025 using the Bornhuetter-Ferguson method.

Given All data is as of Dec 31, 2025

EP for CY	2024	1,400
ECR for AY	2024	78%
EP for CY	2025	1,300
ECR for AY	2025	83%

paid loss @	12 mths	590
reported loss @	12 mths	810
reported CDF for	12-ult	1.90
paid loss @	24 mths	980
reported loss @	24 mths	1,300
reported CDF for	24-ult	1.60

Step 1	ECR ultimate	=	ECR	x	EP	
		=	83%	x	1,300	
		=	1,079			

Step 2a The CDF we need is: 1.90 *(reported CDF for 12-ult)*

Step 2b reported BF ultimate

=	rptd loss @ 12 mths	+	(1 - 1/CDF)	x	ECR ultimate
=	810	+	0.4737	x	1,079
=	1,321				

Final Answer: ultimate for AY 2025 = 1,321

Recall:

unpaid	=	ultimate	-	paid
IBNR	=	ultimate	-	reported

Reading: Friedland 10 (Cape Cod Method)
Model: 2018.Spring #18
Problem Type: Reserving Methods - CC with Pure Premium Trend Adjustments

Find Calculate the ultimate for AY 2024 using the reported Cape Cod method.

Given

AY	rptd clms	CDF	pure premium trend
2023	8,000	2.50	1.082
2024	4,900	3.20	1.040
2025	2,500	5.60	1.000

CY	EP
2023	24,000
2024	22,540
2025	21,250

** no rate level adjustments*
** no premium trend*

Step 1 calculate the adjusted reported losses *(do not apply CDFs to the losses - we are not calculating the ultimate here)*

AY	rptd clms	x	PP trend	=	adjusted reported claims
2023	7,600	x	1.049	=	7,969
2024	4,600	x	1.024	=	4,710
2025	2,300	x	1.000	=	2,300
					14,980

Step 2 calculate the used-up-premium (UUP)

CY/AY	EP	/	CDF	=	UUP
2023	20,520	/	2.30	=	8,922
2024	21,160	/	4.50	=	4,702
2025	17,710	/	5.80	=	3,053
					16,677

Step 3 calculate the adjusted ECR at the level: 2023 *(you must back out the pure premium trend)*

ECR	=	Step 1	/	Step 2	/	(PP trend for AY 2023)
	=	**14,980**	/	**16,677**	/	1.049
	=	**85.66%**				

Step 4 calculate the ultimate using the data for: 2023

ultimate	=	rptd clms	+	(1 -	1 /	CDF)	x	ECR	x	EP
	=	7,600	+	(1 -	1 /	2.3)	x	**85.66%**	x	20,520
	=	17,535	<== *final answer*							

Reading: Friedland 11 (Frequency-Severity Methods)
Model: 2019.Spring #15
Problem Type: Reserving Methods - FS (Basic Method)

Find Use a frequency-severity method to estimate the unpaid claims for AY 2025

Given paid loss @ Dec 31, 2025 700

	Cumulative Reported Counts **(CRC)**			
	12	24	36	48
2022	250	238	245	260
2023	275	270	278	
2024	323	320		
2025	375			

	Cumulative Reported Loss ($000s) **(CRL)**			
AY	12	24	36	48
2022	1,250	1,280	1,325	1,430
2023	1,365	1,395	1,450	
2024	1,625	1,675		
2025	1,900			

** no development past 48 months*

Step 1 develop reported counts to ultimate

=====> **link ratios for reported count triangle**

AY	12-24	24-36	36-48	48-ult
2022	0.952	1.029	1.061	
2023	0.982	1.030		
2024	0.991			
2025				
selected	0.975	1.030	1.061	**1.000**

Tail Factor: The triangle is fully developed as of 48 months. That means the 48-ult tail factor is equal to 1.0

=====> **calculate age-to-ultimate LDFs**

	12-ult	24-ult	36-ult	48-ult
age -> ult	1.065	1.093	1.061	**1.000**

<====== (**selected**) x (prior [age -> ult])

(calculate from right-to-left)

=====> **calculate ultimate counts based on latest reported counts**

	'25@12	'24@24	'23@36	'22@48
diagonal	375	320	278	260
ultimate	399	350	295	260

<====== (**diagonal**) x (age -> ult)

Step 2a calculate CRS triangle *(CRS = Cumulative Reported Severity)*

	Cumulative Reported Severity **(CRS)**			
AY	12	24	36	48
2022	5,000	5,378	5,408	5,500
2023	4,964	5,167	5,216	
2024	5,031	5,234		
2025	5,067			

Step 2b develop reported severities to ultimate

=====> **link ratios for reported severity triangle**

AY	12-24	24-36	36-48	48-ult
2022	1.076	1.006	1.017	
2023	1.041	1.010		
2024	1.040			
2025				
selected	1.052	1.008	1.017	**1.000**

Tail Factor: The triangle is fully developed as of 48 months. That means the 48-ult tail factor is equal to 1.0

=====> **calculate age-to-ultimate LDFs**

	12-ult	24-ult	36-ult	48-ult
age -> ult	1.078	1.025	1.017	**1.000**

<====== (**selected**) x (prior [age -> ult])

(calculate from right-to-left)

=====> **calculate ultimate severities based on latest reported severities**

	'25@12	'24@24	'23@36	'22@48
diagonal	5,067	5,234	5,216	5,500
ultimate	5,463	5,363	5,304	5,500

<====== (**diagonal**) x (age -> ult)

Step 3a calculate ultimate losses as: (ultimate counts) x (ultimate severities)

ultimate counts	399	350	295	260	<====== from Step 1
ultimate severities	5,463	5,363	5,304	5,500	<====== from Step 2b
ultimate losses (000s)	2,182	1,875	1,565	1,430	

Step 3b calculate unpaid losses for AY 2025 *<==== pay attention to whether the question asks for **ultimate** or **unpaid***

unpaid	=	ultimate	-	latest paid
	=	2,182	-	700
	=	1,482	*<-- final answer*	

Reading: Friedland 12 (Case Outstanding Methods)
Model: Basic Practice
Problem Type: Reserving Methods - COS (Case O/S Method #2 - Formula)

Reserving-B: Case OS Method (Problem)

Find Calculate the unpaid claim estimate for this self-insured entity for AY 2024

Given Information as of Dec 31, 2025:

case O/S for AY 2024	1,800

<====== company data

industry data	paid	reported
12 - ultimate CDF	6.25	3.13
24 - ultimate CDF	1.86	1.37
36 - ultimate CDF	1.35	1.21
48 - ultimate CDF	1.24	1.08
60 - ultimate CDF	1.16	1.05
72 - ultimate CDF	1.09	1.03

Step 1 highlight the correct CDFs for the given AY

AY	industry CDFs	paid	reported	
2020	72 - ultimate	1.09	1.03	
2021	60 - ultimate	1.16	1.05	
2022	48 - ultimate	1.24	1.08	
2023	36 - ultimate	1.35	1.21	
2024	24 - ultimate	1.86	1.37	<===== use these CDFs
2025	12 - ultimate	6.25	3.13	

Step 2 subsitute the correct CDFs into the case O/S formula for estimating unpaid for AY 2024

case O/S factor	=	1 +	[(rptd:CDF - 1) x paid:CDF]	/	(paid:CDF - rptd:CDF)
	=	1 +	[(1.37 -1) x 1.86]	/	(1.86 - 1.37)
	=	1 +	0.6882	/	0.4900
	=	2.4045			

unpaid	=	case O/S amount	x	case O/S factor
	=	1,800	x	2.4045
	=	4,328		

(final answer)

Reading: Friedland 13 (Berquist-Sherman Methods)
Model: 2019.Fall #21
Problem Type: Reserving Methods - RBS (Reported Berq-Sherm)

Reserving-B: RBS Method (Problem)

Find Use the reported Berquist-Sherman method to calculate the unpaid for AY 2025

Given

	Cumulative Paid Loss ($000s) (CPL)			
AY	12	24	36	48
2022	560	1,325	1,650	1,680
2023	650	1,350	1,720	
2024	615	1,305		
2025	625			

	Cumulative Reported Loss ($000s) (CRL)			
AY	12	24	36	48
2022	1,100	1,650	1,675	1,680
2023	1,250	1,680	1,750	
2024	1,200	1,800		
2025	1,500			

	Cumulative Paid Counts (CPC)			
AY	12	24	36	48
2022	78	106	114	115
2023	80	111	118	
2024	75	99		
2025	82			

	Cumulative Reported Counts (CRC)			
AY	12	24	36	48
2022	108	115	115	115
2023	112	120	120	
2024	104	110		
2025	106			

annual severity trend	5.0%

Step 1a calculate Average Paid Loss (APL) & Average Case O/S (ACOS)

AY	Average Paid Loss ($000s) (APL) 12	24	36	48
2022	7.2	12.5	14.5	14.6
2023	8.1	12.2	14.6	
2024	8.2	13.2		
2025	7.6			

Ex: 8.1 = 650 / 80
APL = CPL / CPC

AY	Average Case O/S ($000s) (ACOS) 12	24	36	48
2022	18.0	36.1	25.0	0.0
2023	18.8	36.7	15.0	
2024	20.2	45.0		
2025	36.5			

Ex: 18.8 = (1250 - 650) / (112 - 80)
ACOS = (CRL - CPL) / (CRC - CPC)

Step 1b calculate & select severity trend down columns *(if trend is not provided)*

AY	Average Paid Loss **TREND** 12	24	36	48
2022				
2023	12.5%	-2.4%	0.7%	
2024	1.2%	8.2%		
2025	-7.3%			

severity trend selection: n/a
severity trend given: 5.0%

AY	Average Case O/S **TREND** 12	24	36	48
2022				
2023	4.4%	1.7%	-40.0%	
2024	7.4%	22.6%		
2025	80.7%			

Note increase along latest diagonal.
(Evidence for case reserve strengthening in CY 2025)

Step 2a restate ACOS by de-trending from latest diagonal of original ACOS

AY	Average Case O/S (ACOS) 12	24	36	48
2022	31.5	40.8	14.3	0.0
2023	33.1	42.9	15.0	
2024	34.8	45.0		
2025	36.5			

** latest diagonal doesn't change under de-trending*

Ex: 33.1 = 36.5 / 1.05^2

Step 2b restate CRL using restated ACOS from step 2a: (restated ACOS) x (open counts) + (original CPL)

AY	Restated **CRL** 12	24	36	48
2022	1,505	1,692	1,664	1,680
2023	1,709	1,736	1,750	
2024	1,624	1,800		
2025	1,500			

Ex: 1,709 = 33.1 x (112 - 80) + 650

Step 3b apply development method to restated CRL from step 2b

LDFs

AY	12-24	24-36	36-48	**48-ult**
2022	1.124	0.984	1.009	
2023	1.016	1.008		
2024	1.108			
2025				
selected	1.083	0.996	1.009	**1.000**

Tail Factor: Set = 1.0 if triangle is fully developed.
If trianlge is not fully developed then select something different from 1.0

CDFs

	12-ult	24-ult	36-ult	48-ult
age -> ult	1.088	1.005	1.009	**1.000**

<====== (**selected**) x (prior [age -> ult])
(calculate from right-to-left)

Ults.

	'25@12	**'24@24**	**'23@36**	**'22@48**
diagonal	1,500	1,800	1,750	1,680
ultimate	1,632	1,809	1,766	1,680

<====== (**diagonal**) x (age -> ult)

Step 3b unpaid for AY 2025 = 1,007 *<-- final answer*

Reading: Friedland 05 (Triangles)
Model: 2014.Fall #14
Problem Type: Building Triangles from Raw Claims Data

Find Build the **RY (Report Year)** paid & reported claim triangles. (Unless stated otherwise these are cumulative triangles)

Given

			2020		2021		2022	
				ending		ending		ending
			paid in	case O/S	paid in	case O/S	paid in	case O/S
claim id	AY	RY	CY	CY	CY	CY	CY	CY
1	2020	2020	800	800	700	300	600	100
2	2020	2020	700	0	0	0	0	0
3	2020	2020	700	500	500	400	500	200
4	2020	2020	300	600	800	200	300	100
5	2020	2020	900	500	500	400	500	200
6	2021	2021	-	-	900	300	400	200
7	2021	2021	-	-	800	200	400	200
8	2021	2022	-	-	-	-	500	100
9	2022	2022	-	-	-	-	600	0
10	2022	2023	-	-	-	-	-	-
11	2022	2023	-	-	-	-	-	-
12	2022	2023	-	-	-	-	-	-
13	-	-	-	-	-	-	-	-
14	-	-	-	-	-	-	-	-
15	-	-	-	-	-	-	-	-
16	-	-	-	-	-	-	-	-
17	-	-	-	-	-	-	-	-
18	-	-	-	-	-	-	-	-

Step 1 Sum across **RY** (This is simple in Excel but easy to make a mistake with pencil and paper.)

Suggestion: *Use a ruler to draw a horizontal line between one AY and the next.*

		2020		2021		2022	
		paid in	ending case O/S	paid in	ending case O/S	paid in	ending case O/S
AY	RY	CY	CY	CY	CY	CY	CY
all	2020	3,400	2,400	2,500	1,300	1,900	600
all	2021	0	0	1,700	500	800	400
all	2022	0	0	0	0	1,100	100
all	2023	0	0	0	0	0	0

Step 2 Transfer the colour-coded values from Step 1 into these triangles…

incremental PAID claims

RY	12	24	36
2020	3,400	2,500	1,900
2021	1,700	800	
2022	1,100		
2023			

case O/S

RY	12	24	36
2020	2,400	1,300	600
2021	500	400	
2022	100		
2023			

Step 3 (final answer) Form the CUMULATIVE paid triangle by summing across rows in the incremental paid triangle

cumulative PAID claims

RY	12	24	36
2020	3,400	5,900	7,800
2021	1,700	2,500	
2022	1,100		
2023			

Ex: 5,900 = 3,400 + 2,500

7,800 = 3,400 + 2,500 + 1,900

Step 4 (final answer) Form the CUMULATIVE reported triangle: *cumulative reported claims = (cumulative paid claims) + (Case O/S)*

cumulative REPORTED claims

RY	12	24	36
2020	5,800	7,200	8,400
2021	2,200	2,900	
2022	1,200		
2023			

Reading: Friedland 07 (Development Method)
Model: 2017.Fall #19
Problem Type: Reported Claims Development

Find

a Calculate ultimate claims for AY 2022 and 2023 using data as of year-end 2023

b Calculate a diagnostic showing a recent operational change and describe a likely scenario

Given

cumulative reported claims

AY	12	24	36	48
2020	1,440	2,625	3,240	3,476
2021	1,385	2,589	3,003	
2022	1,333	2,556		
2023	1,286			

cumulative paid claims

AY	12	24	36	48
2020	900	2,232	3,070	3,414
2021	878	2,213	2,755	
2022	857	2,195		
2023	837			

reported claims development technique ultimates:

AY	amt
2020	3,600
2021	3,337

a **link ratios for reported claim triangle**

AY	12-24	24-36	36-48	48-ult
2020	1.823	1.234	1.073	
2021	1.870	1.160		
2022	1.917			
2023				
selected	1.870	1.197	1.073	**1.036**

Trick: The triangle is not fully developed as of 48 months. You must calculate a tail factor using AY 2020 information as shown below.

= 3,600 / 3,476

AY 2020 ultimate

AY 12 @48 months

calculate age-to-ultimate LDFs

	12-ult	24-ult	36-ult	48-ult
age -> ult	2.487	1.330	1.111	**1.036**

<====== (**selected**) x (prior [age -> ult])

(calculate from right-to-left)

calculate ultimate losses based on latest reported losses

	'23@12	**'22@24**	**'21@36**	**'20@48**
diagonal	1,286	2,556	3,003	3,476
ultimate	3,198	3,400	3,337	3,600

<====== (**diagonal**) x (age -> ult)

Final answers:

AY 2022 ultimate loss based on reported losses: 3,400

AY 2023 ultimate loss based on reported losses: 3,198

b Since we're only given cumulative reported and paid triangles, we don't have many options for diagnostics. We could calculate incremental triangles, but we'll first try the obvious and calculate **paid / reported**.

cumulative paid / reported

AY	12	24	36	48
2020	0.63	0.85	0.95	0.98
2021	0.63	0.85	0.92	
2022	0.64	0.86		
2023	0.65			

Looking down the columns, we see there has been a general **increase** in the paid/reported ratio

This likely means: *(paid values went up) OR (reported values went down) OR (a combination of both)*

Possible scenarios corresponding to these observations are:

[1] claims handling rules were LOOSENED

[2] case reserve adequacy DECREASED

Reading: Friedland 09 (Bornhuetter-Ferguson Method)
Model: 2019.Spring #19
Problem Type: Reserving Methods - BF with Tort Reform

Reserving-C: BF Method - Tort Reform (Problem)

Find Calculate the ultimate for AY 2024 using:

(i) reported development method
(ii) reported Bornhuetter-Ferguson method

Given Data is for CY/AY 2024 as of Dec 31, 2024

EP	2,000
ECR	87%
reported loss @ 12 months	1,120
reported CDF for 12-ult	2.300
paid loss @ 12 months	460

Tort reform is passed on Dec 31, 2024 causing an expected 20% reduction on future claim payments.

(i) reported development method

Step 1 reported development estimate without tort reform adjustment

=	rptd loss	x	CDF	
=	1,120	x	2.300	
=	2,576			

Step 2 unpaid claims *(need this because we make the tort reform adjustment only to unpaid claims)*

=	ultimate	-	paid loss
=	2,576	-	460
=	**2,116**		

Step 3 adjust unpaid loss downward by: 20%

adjusted unpaid loss

=	**2,116**	x	80%
=	**1,693**		

Step 4 adjusted ultimate

=	adjusted unpaid loss	+	paid loss
=	**1,693**	+	460
=	2,153	<== *final answer*	

(ii) reported Bornhuetter-Ferguson method ** Steps 2,3,4 are the same as for part (i)*

Step 1a ECR method estimate without tort reform adjustment

=	ECR	x	EP
=	87%	+	2,000
=	1,740		

Step 1b BF method estimate without tort reform adjustment

=	rptd loss	+	(1 - 1/CDF)	x	ECR ult
=	1,120	+	0.565	x	1,740
=	2,103				

Step 2 unpaid claims *(need this because we make the tort reform adjustment only to unpaid claims)*

=	ultimate	-	paid loss
=	2,103	-	460
=	**1,643**		

Step 3 adjust unpaid loss downward by: 20%

adjusted unpaid loss

=	**1,643**	x	80%
=	**1,315**		

Step 4 adjusted ultimate

=	adjusted unpaid loss	+	paid loss
=	**1,315**	+	460
=	1,775	<== *final answer*	

Reading: Friedland 10 (Cape Cod Method)
Model: 2018.Spring #18
Problem Type: Reserving Methods - CC (prior AYs) with Pure Premium Trend Adjustments

Find Calculate the ultimate for AY 2024 using the reported Cape Cod method.

Given

AY	rptd clms	CDF	pure premium trend
2023	2,900	2.30	1.067
2024	1,800	3.90	0.983
2025	1,000	7.60	1.000

CY	EP
2023	6,500
2024	8,100
2025	8,000

** no rate level adjustments*
** no premium trend*

Step 1 calculate the adjusted reported losses *(do not apply CDFs to the losses - we are not calculating the ultimate here)*

AY	rptd clms	x	PP trend	=	adjusted reported claims
2023	2,900	x	1.067	=	3,094
2024	1,800	x	0.983	=	1,769
2025	1,000	x	1.000	=	1,000
					5,864

Step 2 calculate the used-up-premium (UUP)

CY/AY	EP	/	CDF	=	UUP
2023	6,500	/	2.30	=	2,826
2024	8,100	/	3.90	=	2,077
2025	8,000	/	7.60	=	1,053
					5,956

Step 3 calculate the adjusted ECR at the level: 2024 *(you must back out the pure premium trend)*

ECR	=	Step 1	/	Step 2	/	(PP trend for AY 2024)
	=	**5,864**	/	**5,956**	/	0.983
	=	**100.16%**				

Step 4 calculate the ultimate using the data for: 2024

ultimate	=	rptd clms	+	(1 -	1 /	CDF)	x	ECR	x	EP
	=	1,800	+	(1 -	1 /	3.9)	x	**100.16%**	x	8,100
	=	7,833	<== *final answer*							

Reserving-D: FS Method - Disposal Rates (Problem)

Reading: Friedland 10 (Frequency-Severity Methods)
Model: 2019.Fall #20
Problem Type: Reserving Methods - FS (Disposal Rate Method)

Find Use the frequency-severity disposal rate method to estimate unpaid claims for AY 2025
Data Set:

Given A court ruling on Dec 31, 2025 will increase future claims payments by 20%
All claims are closed by 48 months
Severity trend: 0%

	Cumulative Paid Counts **(CPC)**				Ultimate
AY	12	24	36	48	Counts
2022	308	555	642	647	647
2023	356	563	678		683
2024	358	575			684
2025	402				795

	Cumulative Paid Loss ($000s) **(CPL)**			
AY	12	24	36	48
2022	375	745	906	916
2023	397	750	922	
2024	422	762		
2025	385			

Reserving-D: FS Method - Disposal Rates (Solution)

Step 1a calculate CPC and IPC

(you will be given 1 of them and have to calculate the other)

given	Cumulative Paid Counts **(CPC)**			
AY	12	24	36	48
2022	308	555	642	647
2023	356	563	678	
2024	358	575		
2025	402			

	Incremental Paid Counts **(IPC)**			
AY	12	24	36	48
2022	308	247	87	5
2023	356	207	115	
2024	358	217		
2025	402			

Step 1b calculate CPL and IPL *(you will be given 1 of them and have to calculate the other)*

given	Cumulative Paid Loss ($000s) **(CPL)**			
AY	12	24	36	48
2022	375	745	906	916
2023	397	750	922	
2024	422	762		
2025	385			

	Incremental Paid Loss ($000s) **(IPL)**			
AY	12	24	36	48
2022	375	370	161	10
2023	397	353	172	
2024	422	340		
2025	385			

Step 2a calculate CDR as: CPC / UC *(UC = Ultimate Counts)*

	Claims DIsposal Rate **(CDR)**			
AY	12	24	36	48
2022	0.476	0.858	0.992	1.000
2023	0.521	0.824	0.993	
2024	0.523	0.841		
2025	0.506			
select	**0.507**	**0.841**	**0.993**	**1.000**

<-- default selection = all period average

Step 2b project IPC *(project the Incremental Paid Counts to the lower right portion of the IPC triangle)*

	Incremental Paid Counts **(IPC)**			
AY	12	24	36	48
2022				
2023				5
2024			104	5
2025		266	121	6

This is the part of the calculation where it's easy to make a mistake.

You have to distribute the remaining counts proportionately using the disposal rate you selected in the previous step.

			remaining counts		proportional distribution		
2025/24:	266	=	(795 - 402)	x	(0.841 - 0.507)	/	(1 - 0.507)
2025/36:	121	=	(795 - 402)	x	(0.993 - 0.841)	/	(1 - 0.507)
2025/48:	6	=	(795 - 402)	x	(1 - 0.993)	/	(1 - 0.507)
* 2024/36:	104	=	(684 - 575)	x	(0.993 - 0.841)	/	(1 - 0.841)
* 2024/48:	5	=	(684 - 575)	x	(1 - 0.993)	/	(1 - 0.841)
* 2023/48:	5	=	(683 - 678)	x	(1 - 0.993)	/	(1 - 0.993)

* *Note that don't have to project every AY. You only have to project the rows for AYs you're asked to calculate.*

Step 3a calculate IPS trended to AY 2025, and select an AY 2025 severity for the unpaid periods 24, 36, 48

	Incremental Paid Severity **(IPS)**				
AY	12	24	36	48	
2022	1.218	1.498	1.851	2.000	
2023	1.115	1.705	1.496		
2024	1.179	1.567			
2025	**0.958**	**1.590**	**1.673**	**2.000**	*<-- default selected severity = all period average*
counts		266	121	6	*<-- from Step 2b*
Step 3b unpaid		423	202	12	*<-- (selected severity) x counts*
court ruling adj.		20%	20%	20%	*<-- adjust unpaid by this percentage*
adj. unpaid		508	243	14	765 *<-- final answer for UNPAID loss*
					1,150 *<-- ULTIMATE loss*

demonstration of severity calc & trending for age 24

1.498	=	(370 / 247) x (1 + 0%)^3
1.705	=	(353 / 207) x (1 + 0%)^2
1.567	=	(340 / 217) x (1 + 0%)^1

Reading: Friedland 12 (Case Outstanding Methods)
Model: 2019.Fall #18
Problem Type: Reserving Methods - COS (Case O/S Method #1)

Find: Use a Case O/S method to calculate the unpaid for AY 2025
Data Set: *Book of Triangles Scenario 0: stable data*

Given:

	Cumulative Paid Loss ($000s) **(CPL)**			
AY	12	24	36	48
2022	720	1,800	2,880	3,600
2023	720	1,800	2,880	
2024	720	1,800		
2025	720			

	Case Outstanding ($000s) **(COS)**			
AY	12	24	36	48
2022	1,440	1,080	720	360
2023	1,440	1,080	720	
2024	1,440	1,080		
2025	1,440			

48-ultimate paid loss to prior case O/S factor	1.00

** no development past 60 months*

Step 1 calculate IPL (Incremental Paid Loss)

AY	Incremental Paid Loss ($000s) (IPL) 12	24	36	48
2022	720	1,080	1,080	720
2023	720	1,080	1,080	
2024	720	1,080		
2025	720			

Sometimes you're given the incremental paid loss.
If so, then step 1 is not necessary.

Step 2a calculate & make selections for: (current case) / (prior case)

AY	Current/Prior COS 24/12	36/24	48/36	60/48
2022	0.750	0.667	0.500	
2023	0.750	0.667		
2024	0.750			
2025	**0.750**	**0.667**	**0.500**	**0.000**

Demonstration for AY 2022 calculations:

			current case O/S		prior case O/S
24/12	0.750	=	1,080	/	1,440
36/24	0.667	=	720	/	1,080
48/36	0.500	=	360	/	720

<-- default selection = all period average

Step 2b project case O/S using selections from step 2a

AY	projected Case Outstanding (COS) 12	24	36	48
2022				360
2023			720	360
2024		1,080	720	360
2025	1,440	1,080	720	360

Demonstration for AY 2025 projections:

	projection		selection		prior case O/S
24 mths	1,080	=	**0.750**	x	1,440
36 mths	720	=	**0.667**	x	1,080
48 mths	360	=	**0.500**	x	720

Step 3a calculate & make selections for: (current IPL) / (prior case)

AY	IPL / (prior case) 24/12	36/24	48/36	60/48
2022	0.750	1.000	1.000	
2023	0.750	1.000		
2024	0.750			
2025	**0.750**	**1.000**	**1.000**	

Demonstration for AY 2022 calculations:

			current IPL		prior case O/S
24/12	0.750	=	1,080	/	1,440
36/24	1.000	=	1,080	/	1,080
48/36	1.000	=	720	/	720

<-- default selection = all period average

Step 3b project IPL using selections from step 3a

AY	projected Incremental Paid Loss (IPL) 12	24	36	48	60
2022					360
2023				720	360
2024			1,080	720	360
2025		1,080	1,080	720	360

Demonstration for AY 2025 projections:

	projection		selection *		prior case O/S
24 mths	1,080	=	**0.750**	x	1,440
36 mths	1,080	=	**1.000**	x	1,080
48 mths	720	=	**1.000**	x	720
60 mths	360	=	1.000	x	360

Note: *The IPL projection always has 1 extra column. In this case 60-months. We use the 48-month case O/S to project the IPL out to 60 months.*

** The 60-mth selection was given info. (It was not selected in step 3a)*

Final calculate ultimate & unpaid amounts:

unpaid = sum of rows in step 3b
ultimate = sum of rows in step 3b + last diagonal of **CPL**

AY	ultimate	unpaid
2022	3,960	360
2023	3,960	1,080
2024	3,960	2,160
2025	3,960	3,240

Pay attention to whether the question asks for ultimate or unpaid.
Also, pay attention to which AY is being asked about.

The final unpaid for AY 2025 is 3,240 *<-- final answer*

Reading: Friedland 13 (Berquist-Sherman Methods)
Model: 2016.Spring #20
Problem Type: Reserving Methods - PBS (Paid Berq-Sherm)

Reserving-D: PBS Method (Problem)

Find Use the paid Berquist-Sherman method to calculate the ultimate for AY 2025 using both linear interpolation and exponential regression to restate the cumulative paid losses.

Given

	Cumulative Paid Counts **(CPC)**				Ultimate
AY	12	24	36	48	Counts
2022	3,314	4,260	4,340	4,380	4,380
2023	3,390	4,404	4,550		4,596
2024	3,342	4,365			4,454
2025	3,607				4,509

** Assume no development past 48 mths.*

	Cumulative Paid Loss ($000s) **(CPL)**			
AY	12	24	36	48
2022	7,760	13,664	15,515	16,484
2023	8,797	13,543	16,824	
2024	7,821	13,928		
2025	9,113			

** Assume no development past 48 mths.*

	parameters for 2-point exponential fit: $y = ae^{bx}$									
	0 - 12		12 - 24		24 - 36		36 - 48		48 - 60	
AY	a	b	a	b	a	b	a	b	a	b
2022	use 12 - 24 values		1,069	0.00060	16	0.00159	22	0.00152	not needed	
2023	use 12 - 24 values		2,079	0.00043	19	0.00149				
2024	use 12 - 24 values		1,187	0.00056						
2025	use 12 - 24 values									

** My solution rounds the adjusted counts to the nearest integer for clarity of presentation, but this is not a requirement. It might be better to keep 1 decimal place because the restated losses should then be more accurate.*

Step 1a calculate CDR as: CPC / UC *(UC = Ultimate Counts)*

AY	Claims DIsposal Rate (CDR) 12	24	36	48
2022	0.757	0.973	0.991	**1.000**
2023	0.738	0.958	**0.990**	
2024	0.750	**0.980**		
2025	**0.800**			
last CY	**0.800**	**0.980**	**0.990**	**1.000**

We assume the latest diagonal reflects current disposal rates. (This is the most recent CY.)
We will restate all AYs to be at the level of this most recent CY.

<-- select LATEST DIAGONAL (not all-period-average)

Step 1b restate CPC using CDRs from last CY

This "range" table is just a visual aid

AY	Restated Cumulative Paid Counts (CPC) 12	24	36	48
2025	3,504	4,292	4,336	4,380
2026	3,677	4,504	4,550	
2027	3,563	4,365		
2028	3,607			

===>

AY	Range Relative to Original Paid Counts 12	24	36	48
2022	12-24	24-36	24-36	48-60
2023	12-24	24-36	36-48	
2024	12-24	24-36		
2025	12-24			

Ex: 3,504 = 0.8 x 4380
Ex: 4,504 = 0.98 x 4596

The range table is helpful in visualizing which data points or which regression parameters to use in Step 2.

Step 2 restate CPL using linear interpolation **or...** restate CPL using exponential regression interpolation

AY	Restated Cumulative Paid Loss (CPL) 12	24	36	48
2022	8,946	14,404	15,422	16,484
2023	10,140	15,790	16,824	
2024	9,140	13,928		
2025	9,113			

AY	Restated Cumulative Paid Loss (CPL) 12	24	36	48
2022	8,751	14,719	15,785	16,484
2023	10,105	15,606	16,824	
2024	8,730	13,928		
2025	9,113			

Exs:

8,946 = (3504-3314)/(4260-3314)x(13664-7760)+7760
15,790 = (4504-4404)/(4550-4404)x(16824-13543)+13543
15,422 = (4336-4260)/(4340-4260)x(15515-13664)+13664

8,751 = 1069 x EXP(0.0006 x 3504)
15,606 = 19 x EXP(0.00149 x 4504)
15,785 = 16 x EXP(0.00159 x 4336)

Step 3a apply development method to restated CPL (Linear)

apply development method to restated CPL (Exp)

LDFs

AY	12-24	24-36	36-48	48-ult
2022	1.610	1.071	1.069	
2023	1.557	1.065		
2024	1.524			*tail*
2025				*factor:*
selected	1.564	1.068	1.069	**1.034**

or...

AY	12-24	24-36	36-48	48-ult
2022	1.682	1.072	1.044	
2023	1.544	1.078		
2024	1.595			*tail*
2025				*factor:*
selected	1.607	1.075	1.044	**1.022**

CDFs

	12-24	24-36	36-48	48-ult
age -> ult	1.846	1.180	1.105	**1.034**

	12-24	24-36	36-48	48-ult
age -> ult	1.843	1.147	1.067	**1.022**

Ults.

	'25@12	'24@24	'23@36	'22@48
diagonal	9,113	13,928	16,824	16,484
ultimate	16,821	16,441	18,595	17,043

	'25@12	'24@24	'23@36	'22@48
diagonal	9,113	13,928	16,824	16,484
ultimate	16,793	15,972	17,947	16,843

Step 3b ultimate for AY 2025 = 16,821 *<-- estimate based on LINEAR INTERPOLATION*
ultimate for AY 2025 = 16,793 *<-- estimate based on EXPONENTIAL REGRESSION INTERPOLATION*

Step 3c The estimate of ultimate using exponential regression differs from the estimate using liner interpolation by: -0.17%
(Step 3c was just for fun to see if there is any real difference between linear and exponential interpolation.)

Reading: Friedland 15 (Evaluating Methods)
Model: Emergence Patterns
Problem Type: Calculate emergence in a future period

Find Calculate the emergence of paid claims in the period 39 to 42

Given Here is the current information for a particular AY:

paid	claims	at	36	months	370

12	- ultimate CDF	2.50
24	- ultimate CDF	1.31
36	- ultimate CDF	1.13
48	- ultimate CDF	1.06
60	- ultimate CDF	1.03
72	- ultimate CDF	1.01

Step 1 determine the 12-month period containing the period you're given

period 39 to 42 is contained in period 36 to 48

Step 2 calculate the quantites required in the emergence formula (for step 3)

ultimate	=	paid @36	x	36-ultimate CDF	
	=	370	x	1.13	
	=	**418**			
unpaid @ 36	=	**ultimate**	-	paid @36	
	=	**418**	-	370	
	=	48			
%paid @ 36	=	1	/	36-ultimate CDF	
	=	1	/	1.13	
	=	**88.5%**			
%paid @ 48	=	1	/	48-ultimate CDF	
	=	1	/	1.06	
	=	94.3%			
%unpaid @ 36	=	1	-	%paid @ 36	
	=	1	-	**88.5%**	
	=	11.5%			

Step 3 calculate the emergence in the 12-month period using the results from step 2

12-mth emergence	=	unpaid @ 36	x	(%paid @ 48	-	%paid @ 36)	/	(%unpaid @ 36)
	=	48	x	(94.3%	-	**88.5%**)	/	11.5%
	=	**24.4**						

Step 4 if the given period was NOT a 12-month interval then you must interpolate to get the final answer

given period ==========> 39 to 42 which is 3 months

3-mth emergence	=	**24.4**	x	3	/	12
	=	6.1	<== final answer			

Reading: Friedland 14 (S/S)
Model: 2016.Fall #24(b)
Problem Type: Ratio method (additive/multiplicative) for S/S

Problem Use the multiplicative ratio method to estimate the ultimate S/S for AY 2014

cumulative paid claims:

AY	12	24	36	48
2011	200	560	570	570
2012	150	250	400	
2013	150	350		
2014	50			

cumulative paid SS:

AY	12	24	36	48
2011	20	90	100	100
2012	16	40	70	
2013	15	56		
2014	5			

selected ultimate claims by AY (using paid claim development)

AY	ult clms		
2011	570	<==	*paid claims development method*
2012	400	<==	*paid claims development method*
2013	458	<==	*paid claims development method*
2014	150	<==	*given in the statement of the exam problem*

When selecting development factors, you may select a SIMPLE AVERAGE in this problem.

** There is no development beyond 48 months.*

Step 1: Ratio of *cumulative* paid S/S to *cumulative* paid claims:

AY	12	24	36	48
2011	10.00%	16.07%	17.54%	17.54%
2012	10.67%	16.00%	17.50%	
2013	10.00%	16.00%		
2014	10.00%			

Step 2: Development triangle *(multiplicative):*

AY	12-24	24-36	36-48	48-		
2011	1.6070	1.0915	1.0000		<===	MULTIPLICATIVE development
2012	1.4995	1.0938				
2013	1.6000					
2014						
selected	1.5688	1.0927	1.0000	1.0000	<===	tail = 1.0
cum	1.7142	1.0927	1.0000	1.0000	<===	selected x (previous cum)
Ult Ratio	17.14%	17.48%	17.50%	17.54%	<===	cum x (latest diagonal from Step 1)

AY	2014	2013	2012	2011	<===	given information
ult clms	150	458	400	570		*(repeated here for convenience)*
pd S/S	5	56	70	100		

Step 3: Ultimate S/S = (Ultimate Ratio) x (Ultimate Claims)
Unpaid S/S = (Ultimate S/S) - (Paid S/S)

AY	Ult S/S	UnPd S/S
2011	100.0	(0.0)
2012	70.0	-
2013	80.1	24.1
2014	25.7	20.7
	275.7	44.7

Step 4: ultimate S/S for AY 2014 = 25.7 <=== final answer

*For **COMPARISON**, the S/S ultimate and unpaid values using standard paid S/S development are:*

Development method:

AY	Ult S/S	UnPd S/S
2011	100.0	-
2012	70.0	-
2013	80.1	24.1
2014	25.6	20.6
	275.7	44.7

% Difference: [(ratio method) - (development method)] / (ratio method)

Ult S/S	UnPd S/S
0%	-
0%	-
0%	0%
0%	1%
0%	0%

Reading: Friedland 14 (Reinsurance)
Model: 2017.Fall #25
Problem Type: Apply reinsurance treaties and calculate net unpaid claims

Problem Calculate the net unpaid claims for all accident years using reported claims development.

cumulative gross reported claims:
(no reported claim development beyond 48 months)

AY	12	24	36	48
2013	2,757	5,570	6,880	7,047
2014	2,345	4,104	5,121	
2015	2,639	4,677		
2016	2,802			

cumulative ceded reported claims:
(under excess-of-loss treaty)

AY	12	24	36	48
2013	0	745	1,332	1,332
2014	0	0	402	
2015	154	328		
2016	0			

cumulative net paid claims:
(net of excess-of-loss treaty)

AY	pd clm
2013	5,102
2014	3,834
2015	2,840
2016	1,385

stop-loss limits
(applies AFTER excess-of-loss treaty)

AY	limit
2013	5,000
2014	5,000
2015	5,000
2016	none

There is no further loss development past 48 months

Step 1: Calculate the net reported claims triangle as (gross reported) - (ceded reported)

AY	12	24	36	48
2013	2,757	4,825	5,548	5,715
2014	2,345	4,104	4,719	
2015	2,485	4,349		
2016	2,802			

Step 2: Calculate net ultimate claims using reported development on the net triangle above (net of excess-of-loss)

AY	12-24	24-36	36-48	48-		
2013	1.7501	1.1498	1.0301		<===	MULTIPLICATIVE development
2014	1.7501	1.1499				
2015	1.7501					
2016						
selected	1.7501	1.1499	1.0301	1.0000	<===	tail = 1.0
cum	2.0730	1.1845	1.0301	1.0000	<===	selected x (previous cum)
Ult Clms net of EoL	5,809	5,151	4,861	5,715	<===	cum x (latest diagonal from Step 1)

Step 3: Put the results from Step 2 into a table and apply the given stop-loss limits

AY	Ult Clms net of EoL	stop-loss limit	Ult Clms net of ALL	net paid	net unpaid		
2013	5,715	5,000	5,000	5,102	0	<===	final answer
2014	4,861	5,000	4,861	3,834	1,027	<===	final answer
2015	5,151	5,000	5,000	2,840	2,160	<===	final answer
2016	5,809	none	5,809	1,385	4,424	<===	final answer
TOTAL					7,611		

Reading: Friedland 16 (ALAE)
Model: 2019.Spring #24
Problem Type: Additive or Multiplicative approach for ALAE

Reserving-E: ALAE - Multiplicative (Problem)

Problem Use the multiplicative approach to estimate the ultimate ALAE for AY 2018

cumulative paid claims:

AY	12	24	36	48
2015	3,800	10,640	15,960	17,556
2016	3,900	10,920	15,600	
2017	3,850	11,858		
2018	4,050			

cumulative paid ALAE:

AY	12	24	36	48
2015	77	316	512	571
2016	81	337	517	
2017	75	334		
2018	82			

selected ultimate claims by AY

AY	ult clms
2015	17,500
2016	17,900
2017	17,600
2018	18,500

<=== *sometimes you are not given the ultimate claims*
- you would then have to calculate them using an appropriate method
- see 2016.Spring #23

When selecting development factors, you may select a SIMPLE AVERAGE in this problem.

Step 1: Ratio of *cumulative* paid ALAE to *cumulative* paid claims:

AY	12	24	36	48
2015	2.03%	2.97%	3.21%	3.25%
2016	2.08%	3.09%	3.31%	
2017	1.95%	2.82%		
2018	2.02%			

Step 2: Development triangle *(either additive or multiplicative as appropriate):*

AY	12-24	24-36	36-48	48-		
2015	1.4631	1.0808	1.0125		<===	MULTIPLICATIVE development
2016	1.4856	1.0712				
2017	1.4462					
2018						
selected	1.4650	1.0760	1.0125	1.0000	<===	tail = 0.0 if additive OR 1.0 if multiplicative
cum	1.5961	1.0895	1.0125	1.0000	<===	MULTIPLICATIVE: selected x (previous cum)
Ult Ratio	3.22%	3.07%	3.35%	3.25%	<===	MULTIPLICATIVE: cum x (latest diagonal from Step 1)

AY	2018	2017	2016	2015	<===	given information
ult clms	18,500	17,600	17,900	17,500		*(repeated here for convenience)*
pd ALAE	82	334	517	571		

Step 3:
Ultimate ALAE = (Ultimate Ratio) x (Ultimate Claims)
Unpad ALAE = (Ultimate ALAE) - (Paid ALAE)

AY	Ult ALAE	UnPd ALAE
2015	568.8	(2.3)
2016	599.7	82.7
2017	540.3	206.3
2018	595.7	513.7
	2,304.4	800.4

Step 4: ultimate ALAE for AY 2018 = 595.7 <=== final answer

Reading: Friedland 16 (ALAE)
Model: 2019.Spring #24
Problem Type: Additive or Multiplicative approach for ALAE

Reserving-E: ALAE - Additive (Problem)

Problem Use the additive approach to estimate the ultimate ALAE for AY 2019

cumulative paid claims:

AY	12	24	36	48
2017	4,300	13,900	20,900	22,800
2018	4,500	13,900	21,500	
2019	3,700	11,800		
2020	4,400			

cumulative paid ALAE:

AY	12	24	36	48
2017	116	507	803	866
2018	99	428	719	
2019	89	396		
2020	110			

selected ultimate claims by AY

AY	ult clms
2017	22,800
2018	23,455
2019	19,633
2020	23,209

<=== *sometimes you are not given the ultimate claims*
- you would then have to calculate them using an appropriate method
- see 2016.Spring #23

When selecting development factors, you may select a SIMPLE AVERAGE in this problem.

Step 1: Ratio of *cumulative* paid ALAE to *cumulative* paid claims:

AY	12	24	36	48
2017	2.70%	3.65%	3.84%	3.80%
2018	2.20%	3.08%	3.34%	
2019	2.41%	3.36%		
2020	2.50%			

Step 2: Development triangle *(either additive or multiplicative as appropriate):*

AY	12-24	24-36	36-48	48-		
2017	0.0095	0.0019	-0.0004		<===	ADDITIVE development
2018	0.0088	0.0026				
2019	0.0095					
2020						
selected	0.0093	0.0023	-0.0004	0.0000	<===	tail = 0.0 if additive OR 1.0 if multiplicative
cum	0.0112	0.0019	-0.0004	0.0000	<===	ADDITIVE: selected + (previous cum)
Ult Ratio	3.62%	3.55%	3.30%	3.80%	<===	ADDITIVE: cum + (latest diagonal from Step 1)

AY	2020	2019	2018	2017	<===	given information
ult clms	23,209	19,633	23,455	22,800		*(repeated here for convenience)*
pd ALAE	110	396	719	866		

Step 3:
Ultimate ALAE = (Ultimate Ratio) x (Ultimate Claims)
Unpad ALAE = (Ultimate ALAE) - (Paid ALAE)

AY	Ult ALAE	UnPd ALAE
2017	866.4	0.4
2018	774.0	55.0
2019	697.0	301.0
2020	840.2	730.2
	3,177.6	1,086.6

Step 4: ultimate ALAE for AY 2019 = 697.0 <=== final answer

Reading: Friedland 17 (ULAE)
Model: 2018.Spring #23
Problem Type: Classical/Kittel approach for ULAE

Reserving-E: ULAE - Classical (Problem)

Problem Use the classical approach to estimate the unpaid ULAE for AY 2022

occurrence <== policy type

CY	paid ULAE	paid claims	incurred claims
2018	0	0	0
2019	52,000	350,000	530,000
2020	50,000	415,000	490,000
2021	51,000	496,000	450,000
2022	56,000	384,000	490,000

<== incurred includes reported & IBNR

700	case outstanding *(latest AY)*
400	total IBNR *(latest AY)*
60%	% of total IBNR attributed to future case development on known claims

60%	percent of unallocated work that occurs when a claim is **opened**
40%	percent of unallocated work that occurs when a claim is **closed**

Step 1: classical ULAE ratio ==> (paid ULAE) / (paid claims)

CY	paid ULAE	paid claims	ULAE ratio
2018	0	0	
2019	44,000	332,000	13.3%
2020	54,000	365,000	14.8%
2021	47,000	466,000	10.1%
2022	53,000	333,000	15.9%
			10.0%

<== selected (average)

Sometimes there is a trend in ULAE ratios.
If so, you may need to use judgement
instead of just selecting the average

Step 2: apply formula for unpaid ULAE

unpaid ULAE = (ULAE ratio) x [40% x (Case + IBNER) + 100% x IBNYR]

where:

IBNER = 60% x Total IBNR <== Incurred But Not ENOUGH Reported
= 60% x 400
= 240

IBNYR = Tot IBNR - IBNER <== Incurred But Not YET Reported
= 400 - 240
= 160

therefore:

unpaid ULAE = 10.0% x [40% x 940 + 100% x 160]

unpaid ULAE = 54 <== **final answer**

Reading: Friedland 17 (ULAE)
Model: 2019.Fall #24
Problem Type: Kittel approach for ULAE

Reserving-E: ULAE - Kittel (Problem)

Problem Use the Kittel approach to estimate the unpaid ULAE at CY 2024 year-end

55% <== Expected Clams Ratio (ECR)
claims-made <== policy type

CY	paid claims	incurred claims	paid ULAE
2021	16,591	32,700	1,991
2022	16,400	35,800	1,825
2023	18,100	34,500	1,825
2024	17,100	32,400	1,825

Report Year	earned premium	paid claims	reported claims	% unrpt'd
2021	68,000	20,900	28,700	12.0%
2022	65,900	12,300	23,500	21.6%
2023	66,700	7,300	19,000	36.7%
2024	64,000	4,200	17,100	68.4%

Step 1: Kittel ULAE ratio ==> (paid ULAE) / AVG [(paid claims) , (incurred claims)]

CY	paid ULAE	average (pd, inc)	ULAE ratio
2021	1,991	24,646	8.1%
2022	1,825	26,100	7.0%
2023	1,825	26,300	6.9%
2024	1,825	24,750	7.4%
			7.4%

<== selected (average)

Sometimes there is a trend in ULAE ratios. If so, you may need to use judgement instead of just selecting the average

Step 2a calculate ultimate (use Bornhuetter-Ferguson reported method)

(usually, we are given case O/S and IBNR in these types of problems but here we have to calculate it ourselves)

AY	ultimate	=	reported	+	%unrptd	x	ECR	x	EP
2021	33,188	=	28,700	+	12.0%	x	55%	x	68,000
2022	31,329	=	23,500	+	21.6%	x	55%	x	65,900
2023	32,463	=	19,000	+	36.7%	x	55%	x	66,700
2024	41,177	=	17,100	+	68.4%	x	55%	x	64,000
	138,157		88,300						

Step 2b calculate case O/S and IBNR in total for all years

case O/S	=	reported	-	paid	=	88,300	-	44,700	=	43,600
IBNR	=	ultimate	-	reported	=	138,157	-	88,300	=	49,857

Trick: Since these policies are **claims-made**, there is no pure IBNR (no IBNYR).

total IBNR	=	IBNER	+	IBNYR
49,857	=	IBNER	+	0

Therefore IBNER = 49,857

Step 3 apply formula for unpaid ULAE

unpd ULAE	=	ULAE ratio	x [50% x (	case	+	IBNER	)	+ 100% x	IBYNR
	=	7.4%	x [50% x (	43,600	+	49,857	)	+ 100% x	0
	=	7.4%	x	46,729					
	=	3,458							

(final answer)

Note: Part (b) of this exam problem asked how the calculations would change if the policies were "occurrence" instead of claims-made.

The answer is that "occurrence" policies have pure IBNR (IBYNR). You could use the same formula as in Step 3, but you would need a way of separating the total IBNR into its components IBNER and IBNYR.

Reading: Friedland 17 (ULAE)
Model: ULAE Example
Problem Type: Congor & Nolibos approach for ULAE

Find Estimate the unpaid ULAE as of the latest given year-end using all 3 versions of the Congor and Nobilos approach.

Given independent estimate of ultimate claims for all AYs 460,000

% of ULAE spend on opening claims	55%
% of ULAE spend on maintaining claims	30%
% of ULAE spend on closing claims	15%

CY	paid ULAE in CY M	ultimate for clms reported in CY R	paid loss in CY P	ultimate for clms closed in CY C
2021	5,000	57,500	22,500	21,500
2022	5,000	70,200	28,400	28,000
2023	5,500	84,900	34,600	35,600
2024	5,500	106,100	43,300	44,100
2025	5,900	131,600	52,000	57,300

Step 1a calculate the claims basis B

CY	55%	x	R	+	30%	x	P	+	15%	x	C	claims basis B
2021	55%	x	57,500	+	30%	x	22,500	+	15%	x	21,500	41,600
2022	55%	x	70,200	+	30%	x	28,400	+	15%	x	28,000	51,330
2023	55%	x	84,900	+	30%	x	34,600	+	15%	x	35,600	62,415
2024	55%	x	106,100	+	30%	x	43,300	+	15%	x	44,100	77,960
2025	55%	x	131,600	+	30%	x	52,000	+	15%	x	57,300	96,575
			450,300				180,800				186,500	**329,880**

Step 1b select a ULAE ratio W: W = M / B

CY	paid ULAE M	claims basis B	ULAE ratio W
2021	5,000	41,600	12.0%
2022	5,000	51,330	9.7%
2023	5,500	62,415	8.8%
2024	5,500	77,960	7.1%
2025	5,900	96,575	6.1%
	26,900	**329,880**	8.154%

If you decided at the beginning you are going to use a weighted average for B then you only need the totals line in Step 1a and 1b. The problem with that however is that you may not see trends in the ULAE ratio.

<====== *you can select this weighted average or choose another reasonable ratio*

Step 2a calculate the estimate of unpaid ULAE using the Expected Claims approach

unpd ULAE	=	W	x	L	-	M	
	=	8.154%	x	460,000	-	26,900	
	=	10,608					

Step 2b calculate the estimate of unpaid ULAE using the Bornhuetter-Ferguson approach

unpd ULAE	=	W	x	(	L	-	B	)
	=	8.154%	x	(	460,000	-	**329,880**	)
	=	10,610						

Step 2b calculate the estimate of unpaid ULAE using the Development approach

unpd ULAE	=	M	x	(	L	/	B	-	1	)
	=	26,900	x	(	460,000	/	**329,880**	-	1	)
	=	10,611								

Notes: *You get the same answer for all 3 methods if your selection for W is the weighted average. You also have to keep enough decimal places or they may differ due to rounding.*

If you choose something other than the weighted average, which could be entirely reasonable depending on the circumstances then you won't in general get the same answer.